

**Units:** SI system: kg (kilogram), m (meter), s (second), C (coulomb);  $\mathcal{A}$  (ampere)=C/s, V (volt)=J/C, F (farad)=C/V,  $\Omega$  (ohm)=V/ $\mathcal{A}$ . 1 mm =  $10^{-3}$  m, 1 cm =  $10^{-2}$  m, 1 km =  $10^3$  m; 1 N (newton)=kg·m/s<sup>2</sup>, 1 J (joule)=N · m = kg · m<sup>2</sup>/s<sup>2</sup>, 1 W (watt)=J/s; prefixes: m (milli)  $10^{-3}$ ,  $\mu$  (micro)  $10^{-6}$ , n (nano)  $10^{-9}$ , p(pico)  $10^{-12}$ , k (kilo)  $10^3$ , M (mega)  $10^6$ .

**Constants:**  $g = 9.8 \text{ m/s}^2$ ,  $k_e = 9 * 10^9 \text{ N m}^2/\text{C}^2 = 1/(4\pi\epsilon_0)$ ,  $\epsilon_0 = 8.85 * 10^{-12} \text{ C}^2/(\text{N m}^2)$ ,  $\mu_0 = 4\pi * 10^{-7} \text{ T-m/A}$ .  $e = -1.6 * 10^{-19} \text{ C}$ ,  $m_e = 9.11 * 10^{-31} \text{ kg}$ ,  $m_p \simeq m_n = 1.67 * 10^{-27} \text{ kg}$

**Volumes.** Cylinder:  $\pi R^2 h$ , sphere:  $\frac{4}{3}\pi R^3$ , cone:  $\frac{1}{3}\pi R^2 h$ . **Areas:** Sphere:  $4\pi R^2$ , circle:  $\pi R^2$ .

**Quadratic equation.**  $ax^2 + bx + c = 0$ ,  $x = \left(-b \pm \sqrt{b^2 - 4ac}\right) / (2a)$

**Derivatives/integrals.**  $\frac{d}{dx}x^n = nx^{n-1}$ ,  $\frac{d}{dx}\sin x = \cos x$ ,  $\frac{d}{dx}\cos x = -\sin x$ ,  $\frac{d}{dx}e^{-ax} = -ae^{-ax}$ ;  $\int x^n dx = \frac{1}{n+1}x^{n+1}$ ,  $\int dx/x = \ln x$ ;  $\int dx / \sqrt{a^2 + x^2} = \ln\left(x + \sqrt{a^2 + x^2}\right)$ ;  $\int dx (a^2 + x^2)^{-\frac{3}{2}} = x / (a^2\sqrt{a^2 + x^2})$ ;  $\int x dx (a^2 + x^2)^{-\frac{3}{2}} = -1 / (\sqrt{a^2 + x^2})$ ;  $\int \sin x dx = -\cos x$ ;  $\int \cos x dx = \sin x$ .

**Vectors.** If  $\vec{c} = \vec{a} + \vec{b}$ , then  $c_x = a_x + b_x$ ,  $c_y = a_y + b_y$ ,  $c_z = a_z + b_z$  and  $c = \sqrt{c_x^2 + c_y^2 + c_z^2}$ . Dot product:  $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = a * b * \cos \theta$ . Cross product:  $\hat{i} \times \hat{j} = \hat{k}$ ,  $\hat{j} \times \hat{k} = \hat{i}$ ,  $\hat{k} \times \hat{i} = \hat{j}$ ;  $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$ ;  $|\vec{A} \times \vec{B}| = A * B * \sin \alpha$ .

**Coulombs Law:**  $F = k_e \frac{q_1 q_2}{r^2}$ ,  $r$ -distance between charges; in vector form  $\vec{F} = k_e \frac{q_1 q_2}{r^2} \hat{r}$ ,  $\hat{r} = \vec{r}/r$  - unit vector from charge  $q_1$  to  $q_2$ ,  $k_e = 9 * 10^9 \dots$  Superposition: if charge  $q_1$  acts on  $q_0$  with  $\vec{F}_{01}$ , charge  $q_2$  acts on  $q_0$  with  $\vec{F}_{02}$ , etc., then  $\vec{F}_{\text{net on } q_0} = \vec{F}_{01} + \vec{F}_{02} + \dots$

**Electric field.** Definition:  $\vec{E} = \vec{F}_0/q_0$  ( charge  $q_0$  is a "probe"). Field from a charge  $q$ :  $E = k_e \frac{q}{r^2}$ ,  $r$ -distance between charge and observation point; in vector form  $\vec{E} = k_e \frac{q}{r^2} \hat{r}$ ,  $\hat{r} = \vec{r}/r$  - unit vector from charge  $q$  to the observation point,  $k_e = 9 * 10^9 \dots$  Superposition: consider charges  $q_1, q_2$ , etc. and the observation point. If  $q_1$  creates field  $\vec{E}_1$  at the observation point,  $q_2$  creates field  $\vec{E}_2$ , etc., then  $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$   
Force on a charge placed in external field  $\vec{E}$ :  $\vec{F} = q\vec{E}$ .

**Gauss Law.** Flux through a small area  $A$  with  $\vec{A}$  along the normal to the surface:  $\Phi = \vec{E} \cdot \vec{A}$ . Flux through a closed surface:  $\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\epsilon_0$ . Field  $E(r)$  from a uniformly charged spherical shell with radius  $R$  and charge  $Q$ :  $E(r < R) = 0$ ,  $E(r > R) = k_e Q/r^2$ . Field  $E(r)$  from a uniformly charged infinite line with linear charge density  $\lambda$ :  $E(r) = \lambda/(2\pi\epsilon_0 r) = 2k_e \lambda/r$ . Field  $E$  from a uniformly charged infinite non-conducting plane with surface charge density  $\sigma$ :  $E = \sigma/(2\epsilon_0) = 2\pi k_e \sigma$ .

**Potential.** Definition:  $V = U/q_0$ . Work:  $W_{AB} = U_A - U_B = -q\Delta V$ . Two charges:  $U = k_e q_1 q_2 / r$ . Point charge:  $V(r) = k_e q / r$ . Superposition:  $V(\vec{r}) = k_e (q_1/r_1 + q_2/r_2 + \dots)$  (with  $r_1$  - distance from  $q_1$  to observation point, etc.). Potential from field:  $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$ ; for uniform field:  $\Delta V = -Ed$ . Field from potential:  $E_x = -dV/dx, \dots$  Conducting sphere with charge  $Q$  and radius  $R$ :  $V(r) = k_e Q/r$ ,  $r \geq R$  and  $V(r) = k_e Q/R$ ,  $r \leq R$ .

**Conductors.** Inside the body of a conductor:  $\vec{E} = 0$ ,  $V = \text{const}$ , no charge. Extra charge - goes to outer surface. Field near the surface of a conductor, *outside*, is  $E = \sigma/\epsilon_0$ ; potential - continuous. Inner surfaces (surfaces of cavities) - uncharged, unless an extra charge is placed inside a cavity.

**Capacitors.** Definition:  $C = Q/V$ . Single sphere:  $C = 4\pi\epsilon_0 R$ . Spherical:  $C = 4\pi\epsilon_0 R_1 R_2 / (R_2 - R_1)$ . Parallel plate capacitor:  $C = A\epsilon_0\kappa/d$ . Energy:  $U_C = Q^2/(2C) = V^2 C/2$ . Connections: parallel (same voltage)  $C_{eq} = C_1 + C_2 + \dots$ ; series (same charge)  $1/C_{eq} = 1/C_1 + 1/C_2 + \dots$  or  $C_{eq} = C_1 C_2 / (C_1 + C_2)$  (for two only).  $Q_{tot} = V C_{eq}$ .

**Current.** Definitions: current  $I = \Delta q/\Delta t \simeq dq/dt$ , density of current  $J = I/A$ , with  $A$  - cross-sectional area. Ohm's law:  $I = V/R$  with  $R$ -resistance. For wire of length  $L$ :  $R = \rho L/A$ , with  $\rho$  - resistivity of material.  $J = E/\rho$ . Power:  $P = IV = I^2 R = V^2/R$ . Simple connections: series (same current)  $R_{eq} = R_1 + R_2 + \dots$ ; parallel (same voltage)  $1/R_{eq} = 1/R_1 + 1/R_2 + \dots$  or  $R_{eq} = R_1 R_2 / (R_1 + R_2)$  (for two only). Microscopic picture:  $J = env_d$ .

**Multiloop circuits and Kirchoff's equations.** Potential changes:  $+\mathcal{E}$  when crossing battery from negative to positive terminal;  $-IR$  when traveling through a resistor along the current. Loop equation: potential drop around a close loop is zero. Junction equation: sum of incoming currents equals sum of outgoing currents.

Charging a capacitor:  $q(t) = \mathcal{E}C(1 - e^{-t/\tau})$  with  $\tau = RC$ ;  $V_C(t) = \frac{q}{C} = \mathcal{E}(1 - e^{-t/\tau})$ ,  $i(t) = i_{max}e^{-t/\tau}$  with  $i_{max} = \frac{\mathcal{E}}{R}$ . Discharging a capacitor:  $\tau = RC$ ,  $q(t) = Q_0 e^{-t/\tau}$ ,  $V(t) = V_0 e^{-t/\tau}$  with  $V_0 = \frac{Q_0}{C}$ ;  $i(t) = i_0 e^{-t/\tau}$  with  $i_0 = \frac{Q_0}{RC}$ .

**Magnetic force.** Force on a particle:  $\vec{F}_m = q\vec{v} \times \vec{B}$ . Revolution in magnetic field: radius  $r = mv/(qB)$ , period  $T = 2\pi m/(qB)$ . Force on a wire:  $\vec{F}_w = I\vec{L} \times \vec{B}$ . Interaction between two parallel wires:  $F = \mu_0 I_1 I_2 L / (2\pi d)$  with  $\mu_0 = 4\pi * 10^{-7}$ . Magnetic moment for a coil with  $N$  turns:  $\vec{\mu} = NI\vec{A}$ ; potential energy:  $U = -\vec{\mu} \cdot \vec{B}$ ; torque  $\vec{\tau} = \vec{\mu} \times \vec{B}$ .

**Fields from currents.** Straight wire:  $B = \mu_0 I / (2\pi d)$ . Bio-Savart:  $d\vec{B} = (\mu_0/4\pi) Id\vec{s} \times \vec{r}/r^3$ . At the center of ring current of radius  $R$ :  $B_{ring} = \mu_0 I / (2R)$ ; for a circular arc with angle  $\theta$  (in radians):  $B = B_{ring} \times \theta / (2\pi)$ . Amper's Law:  $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$ . Field in a solenoid:  $B = \mu_0 I n$ , with  $n$  - density of turns.

**Electromagnetic induction (Faraday Law).** EMF  $\mathcal{E} = -d\Phi_B/dt$ .  $\Phi_B \simeq \vec{A} \cdot \vec{B}$ . If area  $A = \text{const}$ ,  $\mathcal{E} = -A dB/dt$ ; if  $B = \text{const}$ ,  $\mathcal{E} = -B dA/dt$ ; moving rod:  $\mathcal{E} = lvB$ . Induced current:  $I_{ind} = \mathcal{E}/R$ . Direction (Lenz rule): the flux due to induced current tries to oppose the changes in the original flux. **Self-induction.** Magnetic flux through a conductor with current  $I$ :  $\Phi = LI$ ,  $L$ -inductance. For a solenoid with length  $l$ , cross-sectional area  $A$  and density of turns  $n$ , one has  $L/l = \mu_0 A n^2$ . Self-induced EMF:  $\mathcal{E}_L = -L dI/dt$ . Energy:  $U_L = \frac{1}{2} LI^2$ .

**RL circuits.** Time constant  $\tau_L = L/R$ . Decay of current  $I(t) = I_0 \exp(-t/\tau_L)$ . Build up of current  $I(t) = \mathcal{E}/R * [1 - \exp(-t/\tau_L)]$ . At  $t = 0$  inductor acts as infinite resistance; at  $t \rightarrow \infty$  acts as a piece of wire.

**LC and driven LRC circuits.** Free oscillations in LC circuit: natural (or resonant) frequency  $\omega_0 = 1/\sqrt{LC}$  (in rad/s) or  $f_0 = \omega_0/(2\pi)$  (in Hz). Energy conservation:  $\frac{1}{2}q^2/C + \frac{1}{2}Li^2 = \text{const}$  or  $\frac{1}{2}Q_{max}^2/C = \frac{1}{2}LI_{max}^2$ . **Driven:**  $f_d$  (in Hz) - driving frequency;  $\omega_d = 2\pi f_d$ . Inductive reactance:  $X_L = \omega_d L$ ; capacitive reactance:  $X_C = 1/(\omega_d C)$ . Impedance  $Z = \sqrt{R^2 + (X_L - X_C)^2}$ . Amplitude of current:  $I_m = \mathcal{E}_m/Z$ . Phase angle:  $\tan \phi = (X_L - X_C)/R$ . Resonance:  $\omega_d = \omega_0$ ,  $X_L = X_C$ ,  $Z = R = \text{min}$ ,  $I_m = \mathcal{E}_m/R = \text{max}$ . RMS values:  $I_{RMS} = I_m/\sqrt{2}$ ,  $V_{RMS} = \mathcal{E}_m/\sqrt{2}$ ;  $I_{RMS} = V_{RMS}/Z$ . Power  $P = I_{RMS}^2 R$ . Transformers:  $V_2 = V_1 * N_2/N_1$ ,  $I_2 = I_1 * N_1/N_2$ .

**From Phys 111:**

**Kinematics:**  $v = dx/dt$ ,  $a = dv/dt = d^2x/dt^2$ . Circular motion with constant speed:  $\omega = v/R$ ,  $a_c = v^2/R = \omega^2 R$ , towards center.

**The three Laws of motion:** (1) If  $\vec{F}_{net} = 0$  then  $\vec{v} = const$ ; (2)  $\vec{F}_{net} = m\vec{a}$ ; (3)  $\vec{F}_{21} = -\vec{F}_{12}$

**Work and power.**  $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$ . Power:  $P = W/\Delta t = \vec{F} \cdot \vec{v}$ .

**Kinetic energy and work-energy theorem:**  $K = \frac{1}{2}mv^2$ ,  $\Delta K = W$  where  $W$  is the *net* work (i.e. work by all forces).

**Potential energy.** For conservative forces (with path-independent work) introduce  $U(\vec{r})$  so that  $W_{AB} = U_A - U_B = -\Delta U$ . If *only* conservative forces, then energy conservation:  $K + U = const$ .