Formulas - exams 1 and 2

Constants and units: $g = 9.8\,\text{m/s}^2$, $1\,\text{mm} = 10^{-3}\text{m}$, $1\,\text{cm} = 10^{-2}\,\text{m}$, $1\,\text{km} = 10^3\,\text{m}$, $1\,\text{in} = 2.54\,\text{cm}$, $1\,\text{mi} = 1609\,\text{m}$; $1\,\text{N}\,(\text{newton}) = \text{kg}\cdot\text{m/s}^2$, $1\,\text{J}\,(\text{joule}) = \text{N}\cdot\text{m} = \text{kg}\cdot\text{m}^2/\text{s}^2$, 1 W (watt)=J/s. density=mass/volume.

Volumes. Cylinder: $\pi R^2 h$, sphere: $\frac{4}{3}\pi R^3$, cone: $\frac{1}{3}\pi R^2 h$

Quadratic equation. $ax^2 + bx + c = 0\,,\ x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a)$

Derivatives/integrals. $\frac{d}{dt}t^n = nt^{n-1}\,,\ \int r^n\,dr = \frac{1}{n+1}r^{n+1}$

Vectors. If $\vec{c} = \vec{a} + \vec{b}$, then $c_x = a_x + b_x\,,\ c_y = a_y + b_y\,,\ c_z = a_z + b_z$ and $c = \sqrt{c_x^2 + c_y^2 + c_z^2}$. Dot product: $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\alpha$. Cross product: $\hat{\imath}\times\hat{\jmath} = \hat{k}\,,\ \hat{\jmath}\times\hat{k} = \hat{\imath}\,,\ \hat{k}\times\hat{\imath} = \hat{\jmath}$.

Kinematics: $v = dx/dt\,,\ a = dv/dt = d^2x/dt^2$. Constant a: $v - v_0 = at$, $x - x_0 = \frac{v_0+v}{2}t = v_0 t + \frac{1}{2}at^2 = \frac{v^2 - v_0^2}{2a}$. Projectile: $v_x = const\,,\ x - x_0 = v_x t\,,\ v_y = v_{0y} - gt\,,\ y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = (v_{0y}^2 - v_y^2)/(2g)$. Range: $\left(v_0^2/g\right) * \sin(2\theta)$

Circular motion with constant speed: $\omega = v/R$, $a_c = v^2/R = \omega^2 R$, towards center.

The three Laws of motion: (1) If $\vec{F}_{net} = 0$ then $\vec{v} = const$; (2) $\vec{F}_{net} = m\vec{a}$; (3) $\vec{F}_{21} = -\vec{F}_{12}$

Specific forces. Gravity: $m\vec{g}$ (down). Normal $\vec{N}$ - perpendicular to surface; tension T - constant along the string. Spring force: $F = -kx$ (k is spring constant).

Friction - parallel to surface; kinetic: $f_k = \mu_k N$; static: $f_s \leq \mu_s N$ with $N = mg$ (horizontal plane) or $N = mg\cos\theta$ (inclined plane).

Inclined plane. Components of gravity: $mg\sin\theta$ (parallel to plane, downhill) and $mg\cos\theta$ (perpendicular to plane). Kinetic friction: $\mu_k mg\cos\theta$ (parallel to plane, opposite to direction of motion).

Centripetal motion: $F_{net} = mv^2/R$; direction of $\vec{F}_{net}$ - towards center of revolution.

Work and power. Constant force $W = \vec{F}\cdot(\vec{r}_2 - \vec{r}_1) = F_x\Delta x + F_y\Delta y + F_z\Delta z$ (or, $W = F\Delta r\cos\alpha$); general: $W_{AB} = \int_A^B \vec{F}\cdot d\vec{r}$. Power: $P = W/\Delta t = \vec{F}\cdot\vec{v}$. Work by specific forces: gravity: $W_g = -mg\Delta y$ (and Δx does not matter); normal: $W_N = 0$; kinetic friction: $W_f = -fL$; spring $W_s = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$

Kinetic energy and work-energy theorem: $K = \frac{1}{2}mv^2\,,\ \Delta K = W$ where W is the *net* work (i.e. work by all forces).

Potential energy. For conservative forces (with path-independent work) introduce $U(\vec{r})$ so that $W_{AB} = U_A - U_B = -\Delta U$. For specific forces: gravity: $U_g = mgh$; spring: $U_s = \frac{1}{2}kx^2$. If *only* conservative forces, then energy conservation: $K + U = const$. If also non-conservative forces (e.g., friction) with work $W_{non-cons}$, then $\Delta(K + U) = W_{non-cons}$