Circular motion with constant speed:
Rotation (kinematics)

Quadratic equation. \( ax^2 + bx + c = 0, \ x = \left( -b \pm \sqrt{b^2 - 4ac} \right) / (2a) \)

Derivatives/integrals. \( \frac{d}{dt}r^n = n t^{n-1}, \int r^n \, dr = \frac{1}{n+1} r^{n+1} \)

Vectors. If \( \vec{c} = \vec{a} + \vec{b} \), then \( c_x = a_x + b_x, \ c_y = a_y + b_y, \ c_z = a_z + b_z \) and \( c = \sqrt{c_x^2 + c_y^2 + c_z^2} \).

Dot product: \( \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab \cos \alpha \). Cross product: \( |\vec{A} \times \vec{B}| = A \times B \times \sin \alpha \)

\( \hat{i} \times \hat{j} = \hat{k}, \ \hat{j} \times \hat{i} = \hat{k}, \ \hat{k} \times \hat{j} = \hat{i} \).

Kinematics: \( v = dx/dt, \ a = dv/dt = d^2x/dt^2 \). Constant \( a: v - v_0 = at, \ x - x_0 = \frac{v_0 + v}{2} t = v_0 t + \frac{1}{2} at^2 = \frac{v^2 - v_0^2}{2a} \).

Projectile: \( v_x = \text{const}, \ x - x_0 = v_x t, \ y - y_0 = v_y t - \frac{1}{2} gt^2 = \frac{(v_0^2 - v_y^2)}{2g} \). Range: \( \left(v_0^2/g\right) \sin(2\theta) \)

Circular motion with constant speed: \( \omega = v/R, \ \vec{a} = v^2/R = \omega^2 \vec{r}, \ \vec{r} \text{ is parallel to} \ \vec{v} \). If \( \vec{a} \perp \vec{r} \), then \( \vec{a} = -F \times \vec{v} \), 

Rotation (kinematics): \( \vec{v} = \vec{r} \times (\vec{a} \times \vec{r}) \) or \( \vec{v} = \vec{r} \times (\vec{a} \times \vec{r}) \).

Work and power. Constant force \( F = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1) = F_x \Delta x + F_y \Delta y + F_z \Delta z \) (or, \( W = F \Delta r \cos \alpha \)). General: \( W_{AB} = \int_{A}^{B} \vec{F} \cdot d\vec{r} \).

Power: \( P = W/\Delta t = \vec{F} \cdot \vec{v} \). Work by specific forces: gravity: \( W_g = -mg \Delta y \) (and \( \Delta x \) does not matter); normal: \( W_N = 0 \);

Kinetic friction: \( W_{f} = -fL \); spring work: \( W_s = \frac{1}{2} k \left( x_f^2 - x_i^2 \right) \)

K.E and W-E theorem: \( K = \frac{1}{2} mv^2, \ \Delta K = W \) where \( W \) is the net work (i.e. work by all forces).

Potential energy. For conservative forces (with path-independent work) introduce \( U(\vec{r}) \) so that \( W_{AB} = U_A - U_B = -\Delta U \). For specific forces: gravity: \( U_g = mgh \); spring: \( U_s = \frac{1}{2} kx^2 \).

If only conservative forces, then energy conservation: \( \Delta K + \Delta U = 0 \) or \( K + U = \text{const} \). If also non-conservative forces (e.g., friction) with work \( W_{\text{non-cons}} \), then \( \Delta (K + U) = W_{\text{non-cons}} \).

Momentum. \( \vec{p} = m \vec{v}, \ \vec{P}_{\text{tot}} = \sum m_i \vec{v}_i \).

Impulse: \( \Delta \vec{p} = F_{av} \Delta t \). Conservation: if \( F_{\text{ext}} = 0 \), then \( \vec{P}_{\text{tot}} = \text{const} \) (e.g., in collisions).

Center-of-mass: \( \vec{R}_{\text{cm}} = \frac{1}{M} \sum m_i \vec{r}_i, \ M = \sum M_i \);

Rotation (kinematics): If \( N \)-number of revolutions, then \( \theta = 2\pi N \).

Angular velocity \( \omega = d\theta/dt \) (in rad/s); angular acceleration \( \alpha = d\omega/dt \) (in rad/s^2).

\( \omega = \frac{\omega_0 + \alpha t}{2}, \ \text{or} \ \theta = \frac{\omega_0^2 - \omega^2}{2\alpha} \), and \( \omega = \omega_0 + \alpha t \). Connection with linear: \( \omega = v/r, \ \alpha = a/r \).

Dynamics: \( K = \frac{1}{2} I \omega^2 \). I-moment of inertia. For point masses: \( I = \sum m_i r_i^2 \) for solid bodies \( I = \int dV \rho r^2 \). Specific I-s: rod (about center) \( ML^2/12 \); rod (about end) \( ML^2/3 \); hoop \( MR^2/2 \); disk \( MR^2/2 \); solid sphere \( \frac{2}{5} MR^2 \); spherical shell \( \frac{4}{3} MR^2 \).

Parallel axes theorem: \( I = I_{CM} + MD^2 \)

Rotation+linear: \( K = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \). If \( \omega = v/R \) (e.g., rolling) \( K = \frac{1}{2} mv^2 (1 + 1/(mR^2)) \)

Formulas - PHYS111