Formulas PHYS 121 - exam 1

Units: SI system: kg (kilogram), m (meter), s (second), C (coulomb); $\mathcal{A}$ (ampere)=C/s, V (volt)=J/C, F (farad)=C/V, Ω (ohm)=V/$\mathcal{A}$. $1\,\mathrm{mm} = 10^{-3}\,\mathrm{m}$, $1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$, $1\,\mathrm{km} = 10^{3}\,\mathrm{m}$; 1 N (newton)=kg·m/s^2, 1 J (joule)=$\mathrm{N} \cdot \mathrm{m} = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$, 1 W (watt)=J/s; prefixes: m (milli) 10^{-3}, μ (micro) 10^{-6}, n (nano) 10^{-9}, p(pico) 10^{-12}, k (kilo) 10^3, M (mega) 10^6.
Constants: $g = 9.8\,\mathrm{m/s}^2$, $k_e = 9 * 10^9\,\mathrm{N\ m^2/C^2} = 1/(4\pi\epsilon_0)$, $\epsilon_0 = 8.85 * 10^{-12}\,\mathrm{C^2/(N\ m^2)}$, $\mu_0 = 4\pi * 10^{-7}\, T{-}m/A$. $e = -1.6 * 10^{-19}\,\mathrm{C}$, $m_e = 9.11 * 10^{-31}\,\mathrm{kg}$, $m_p \simeq m_n = 1.67 * 10^{-27}\, kg$
Volumes. Cylinder: $\pi R^2 h$, sphere: $\frac{4}{3}\pi R^3$, cone: $\frac{1}{3}\pi R^2 h$. **Areas:** Sphere: $4\pi R^2$, circle: πR^2.
Quadratic equation. $ax^2 + bx + c = 0\,,\ x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a)$
Derivatives/integrals. $\frac{d}{dx}x^n = nx^{n-1}\,,\ \frac{d}{dx}\sin x = \cos x\,,\ \frac{d}{dx}\cos x = -\sin x\,,\ \int x^n\,dx = \frac{1}{n+1}x^{n+1}$, $\int dx/x = \ln x$; $\int dx \Big/ \sqrt{a^2 + x^2} = \ln\left(x + \sqrt{a^2 + x^2}\right)$; $\int dx\left(a^2 + x^2\right)^{-\frac{3}{2}} = x/\left(a^2\sqrt{a^2 + x^2}\right)$; $\int x\,dx\left(a^2 + x^2\right)^{-\frac{3}{2}} = -1/\left(\sqrt{a^2 + x^2}\right)$; $\int \sin x\,dx = -\cos x$; $\int \cos x\,dx = \sin x$.
Vectors. If $\vec{c} = \vec{a} + \vec{b}$, then $c_x = a_x + b_x$, $c_y = a_y + b_y$, $c_z = a_z + b_z$ and $c = \sqrt{c_x^2 + c_y^2 + c_z^2}$. Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = a * b * \cos\theta$. Cross product: $\hat{\imath} \times \hat{\jmath} = \hat{k}\,,\ \hat{\jmath} \times \hat{k} = \hat{\imath}\,,\ \hat{k} \times \hat{\imath} = \hat{\jmath}$; $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}; \left|\vec{A} \times \vec{B}\right| = A * B * \sin\alpha$.

Coulombs Law: $F = k_e \frac{q_1 q_2}{r^2}$, r-distance between charges; in vector form $\vec{F} = k_e \frac{q_1 q_2}{r^2}\hat{r}$, $\hat{r} = \vec{r}/r$ - unit vector from charge q_1 to q_2, $k_e = 9 * 10^9$... Superposition: if charge q_1 acts on q_0 with $\vec{F}_{01}$, charge q_2 acts on q_0 with $\vec{F}_{02}$, etc., then $\vec{F}_{\text{net on } q_0} = \vec{F}_{01} + \vec{F}_{02} + \ldots$

Electric field. Definition: $\vec{E} = \vec{F}_0/q_0$ (charge q_0 is a "probe"). Field from a charge q: $E = k_e \frac{q}{r^2}$, r-distance between charge and observation point; in vector form $\vec{E} = k_e \frac{q}{r^2}\hat{r}$, $\hat{r} = \vec{r}/r$ - unit vector from charge q to the observation point, $k_e = 9 * 10^9$... Superposition: consider charges q_1, q_2, etc. and the observation point. If q_1 creates field $\vec{E}_1$ at the observation point, q_2 creates field $\vec{E}_2$, etc., then $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$.
Force on a charge placed in external field $\vec{E}$: $\vec{F} = q\vec{E}$.

Gauss Law. Flux through a small area A with $\vec{A}$ along the normal to the surface: $\Phi = \vec{E} \cdot \vec{A}$. Flux through a closed surface: $\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\epsilon_0$. Field $E(r)$ from a uniformly charged spherical shell with radius R and charge Q: $E(r < R) = 0$, $E(r > R) = k_e Q/r^2$. Field $E(r)$ from a uniformly charged infinite line with linear charge density λ: $E(r) = \lambda/(2\pi\epsilon_0 r) = 2k_e\lambda/r$. Field E from a uniformly charged infinite non-conducting plane with surface charge density σ: $E = \sigma/(2\epsilon_0) = 2\pi k_e \sigma$.

From Phys 111:
Kinematics: $v = dx/dt\,,\ a = dv/dt = d^2x/dt^2$. Circular motion with constant speed: $\omega = v/R$, $a_c = v^2/R = \omega^2 R$, towards center.
The three Laws of motion: (1) If $\vec{F}_{net} = 0$ then $\vec{v} = const$; (2) $\vec{F}_{net} = m\vec{a}$; (3) $\vec{F}_{21} = -\vec{F}_{12}$
Work and power. $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$. Power: $P = W/\Delta t = \vec{F} \cdot \vec{v}$.
Kinetic energy and work-energy theorem: $K = \frac{1}{2}mv^2$, $\Delta K = W$ where W is the *net* work (i.e. work by all forces).
Potential energy. For conservative forces (with path-independent work) introduce $U(\vec{r})$ so that $W_{AB} = U_A - U_B = -\Delta U$. If *only* conservative forces, then energy conservation: $K + U = const$.