

# Formulas PHYS102, Common Exam 3, Fall 2023

## Formulas for Motion in 1-dimension

**Displacement** =  $\Delta x = x_2 - x_1$

**Average velocity**  $v_{avg} = \frac{x_2 - x_1}{t_2 - t_1}$

**(Average speed)** =  $s_{avg} = \frac{\text{(Total distance)}}{\text{(Time change)}}$

**Average acceleration**  $a_{avg} = \frac{v_2 - v_1}{t_2 - t_1}$

**Motion with a constant velocity, v**  $x = x_0 + v t$

**Motion with a constant acceleration "a"**

$v = v_0 + a t$        $x = x_0 + v_0 t + \frac{1}{2} a t^2$

$v^2 - v_0^2 = 2a(x - x_0)$        $x - x_0 = \frac{1}{2} (v_0 + v) t$

**Free fall motion:**

**Motion with a constant acceleration "-g",**  
where  $g = 9.8 \text{ m/s}^2$  (+y direction points up)

$v = v_0 - g t$        $y = y_0 + v_0 t - \frac{1}{2} g t^2$

$v^2 - v_0^2 = -2g(y - y_0)$

## Vector Components vs. magnitude and direction

$$\begin{cases} A_x = A \cos(\theta) & (\text{adjacent}) = (\text{hypotenuse}) \times \cos \theta \\ A_y = A \sin(\theta) & (\text{opposite}) = (\text{hypotenuse}) \times \sin \theta \end{cases}$$

$$\begin{cases} |\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2} \\ \theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) = \arctan \left( \frac{A_y}{A_x} \right) \\ \text{(add/subtract 180 deg if necessary)} \end{cases}$$

## Equations for Projectile Motion

If the initial velocity  $\vec{v}_0 = (v_{0x}, v_{0y})$        $v_{0x} = v_0 \cos \theta_0$   
and the initial position  $\vec{r}_0 = (x_0, y_0)$  are given,  $v_{0y} = v_0 \sin \theta_0$

X-component (horizontal) motion

$v_x = v_{0x}$

$x = x_0 + v_{0x} t$

Range:  $(v_0^2/g) * \sin(2\theta)$

Y-component (vertical) motion  
(+y direction points up)

$v_y = v_{0y} - g t$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$v_y^2 - v_{0y}^2 = -2g(y - y_0)$

$\Delta y = \left( \frac{v_{0y} + v_y}{2} \right) \cdot t$

**Units:**

$1 \text{ cm} = 10^{-2} \text{ m}$  ;  $1 \text{ km} = 1000 \text{ m}$

$1 \text{ g} = 10^{-3} \text{ kg}$

$1 \text{ lt} = 1000 \text{ ml}$ ;  $1000 \text{ lt} = 1 \text{ m}^3$

Newton's 2<sup>nd</sup> law:  $\vec{F}_{net} = m\vec{a}$

Static friction:  $|\vec{f}_S| < |\vec{f}_S^{Max}| = \mu_S |\vec{F}_N|$

Kinetic friction:  $|\vec{f}_k| = \mu_k |\vec{F}_N|$

$\vec{F}_N$  = (normal force)

### Impulse, momentum

Impulse  $\vec{I} = \vec{F}\Delta t$       Momentum  $\vec{p} = m\vec{v}$

Impulse-momentum theorem  $\vec{I}_{net} = \vec{p}_f - \vec{p}_i$

Conservation of momentum for system

If  $F_{net}=0$ , then  $\vec{P}_{net,f} = \vec{P}_{net,i}$

Elastic Collision:  $v_{1i} - v_{2i} = v_{2f} - v_{1f}$

### Uniform circular motion

$$|\vec{a}| = \frac{v^2}{r}$$

### Newton's Universal Law of Gravitation

$$|F_g| = \frac{G m_1 m_2}{r_{12}^2} \quad G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$g = \frac{G M_{Earth}}{R_{Earth}^2}$$

Work-Energy Theorem:  $W_{net} = K_f - K_i$

$$K = \frac{1}{2}mv^2$$

In 1D  $W_F = F\Delta x$

In 2D,  $W_F = |\vec{F}| |\vec{d}| \cos \theta_{F,d}$

$$\equiv \vec{F} \cdot \vec{d} = F_x d_x + F_y d_y$$

$U_g = mgh$        $W_g = -\Delta U_g$        $g = 9.8 \text{ m/s}^2$

Spring:  $F_s = -kx$        $\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2)$

W<sub>spring</sub> =  $-\frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2}k(x_i^2 - x_f^2)$

$$\Delta E_{mech} = \Delta K + \Delta U$$

If the work done by non-conservative forces is zero, the mechanical energy is conserved.

$$E_{mech,f} = E_{mech,i}$$

If the work done by non-conservative forces is not zero, the mechanical energy changes by the amount of the work done by the non-conservative forces.

$$\Delta E_{mech} = E_{mech,f} - E_{mech,i} = W_{non-conservative}$$

Average power  $P_{avg} = \frac{W}{\Delta t}$       1 hp = 746 W

Instantaneous power  $P = |\vec{F}| |\vec{v}| \cos \theta_{F,v}$

Rotation Counter clockwise: +, Clockwise : -

$2\pi$  radians = 360 degree

If N-number of revolutions, then  $\theta = 2\pi N$ .

Angular displacement  $\Delta\theta \equiv \theta_{final} - \theta_{initial}$

Angular velocity  $\omega_{ave} \equiv \frac{\Delta\theta}{\Delta t}$  Angular acceleration  $\alpha_{ave} \equiv \frac{\Delta\omega}{\Delta t}$

T: period f: frequency

$$\omega = 2\pi f = 2\pi/T \quad f = \omega/2\pi$$

$$\Delta s = r\Delta\theta \quad v_T = r\omega \quad a_T = r\alpha \quad a_r = \frac{v_T^2}{r} = r\omega^2$$

Rotation with constant angular acceleration  $\alpha$

$$\omega_f(t) = \omega_0 + \alpha t \quad \theta_f(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2(t) = \omega_0^2 + 2\alpha[\theta_f - \theta_0]$$

Moment of inertia of particles  $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$

Torque

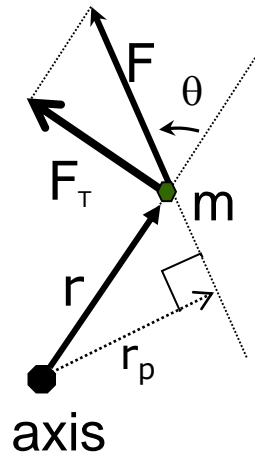
$$|\tau| = rF_T = rF \sin \theta = r_p F$$

$$\tau_{net} = I\alpha$$

Rotation around a fixed axis

$$K = \frac{1}{2} I \omega^2$$

$$U_{gravity} = Mgh_{com}$$

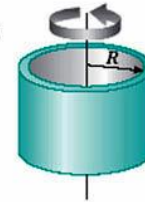


## Moment of inertia of uniform rigid bodies

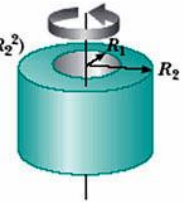
TABLE 10.2

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

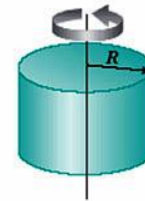
Hoop or thin cylindrical shell  
 $I_{CM} = MR^2$



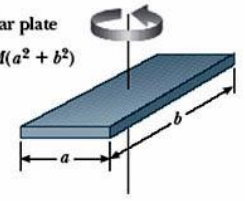
Hollow cylinder  
 $I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$



Solid cylinder or disk  
 $I_{CM} = \frac{1}{2} MR^2$

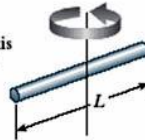


Rectangular plate  
 $I_{CM} = \frac{1}{12} M(a^2 + b^2)$



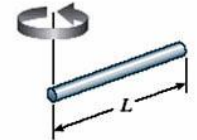
Long, thin rod with rotation axis through center  
 $I_{CM} = \frac{1}{12} ML^2$

$$I_{CM} = \frac{1}{12} ML^2$$

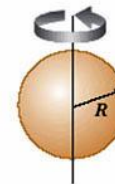


Long, thin rod with rotation axis through end  
 $I = \frac{1}{3} ML^2$

$$I = \frac{1}{3} ML^2$$



Solid sphere  
 $I_{CM} = \frac{2}{5} MR^2$



Thin spherical shell  
 $I_{CM} = \frac{2}{3} MR^2$

$$I_{CM} = \frac{2}{3} MR^2$$

