Formulas PHYS102, Final Exam, Fall 2024

Formulas for Motion in 1-dimension

Displacement = $\Delta x = x_2 - x_1$

Average velocity $v_{avg} = \dfrac{x_2 - x_1}{t_2 - t_1}$

(Average speed) = s_{avg} = $\dfrac{\text{(Total distance)}}{\text{(Time change)}}$

Average acceleration $a_{avg} = \dfrac{v_2 - v_1}{t_2 - t_1}$

Motion with a constant velocity, v $x = x_0 + v\, t$

Motion with a constant acceleration "a"

$$v = v_0 + a\, t \qquad x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 - v_0^2 = 2a(x - x_0) \qquad x - x_0 = \frac{1}{2}(v_0 + v)t$$

Free fall motion:
Motion with a constant acceleration "-g",
where g = 9.8 m/s^2 (+y direction points up)

$$v = v_0 - g\, t \qquad y = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$v^2 - v_0^2 = -2g(y - y_0)$$

Vector Components vs. magnitude and direction

$$\begin{cases} A_x = A\cos(\theta) \\ A_y = A\sin(\theta) \end{cases}$$

$(adjacent) = (hypotenuse) \times \cos\theta$

$(opposite) = (hypotenuse) \times \sin\theta$

$$\begin{cases} |\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2} \\ \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) = \arctan\left(\dfrac{A_y}{A_x}\right) \\ \text{(add/subtract 180 deg if necessary)} \end{cases}$$

Equations for Projectile Motion

If the initial velocity $\vec{v}_0 = (v_{0x}, v_{0y})$ and the initial position $\vec{r}_0 = (x_0, y_0)$ are given,

$$v_{0x} = v_0 \cos\theta_0$$

$$v_{0y} = v_0 \sin\theta_0$$

X-component (horizontal) motion

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x} t$$

$$\text{Range: } (v_0^2/g) * \sin(2\theta)$$

Y-component (vertical) motion (+y direction points up)

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

$$v_y^2 - v_{0y}^2 = -2g(y - y_0)$$

$$\Delta y = \left(\frac{v_{0y} + v_y}{2}\right) \cdot t$$

Units:

$1cm = 10^{-2} m$; 1km=1000m

$1g = 10^{-3} kg$

1 lt =1000ml; 1000 lt =1m^3

Newton's 2nd law: $\vec{F}_{net} = m\vec{a}$

Static friction : $\left|\vec{f}_S\right| < \left|\vec{f}_S^{Max}\right| = \mu_S \left|\vec{F}_N\right|$

Kinetic friction : $\left|\vec{f}_k\right| = \mu_k \left|\vec{F}_N\right|$

$\vec{F}_N$ = (normal force)

Impulse, momentum

Impulse $\vec{I} = \vec{F}\Delta t$ Momentum $\vec{p} = m\vec{v}$

Impulse-momentum theorem $\vec{I}_{net} = \vec{p}_f - \vec{p}_i$

Conservation of momentum for system

If Fnet=0 , then $\vec{P}_{net,f} = \vec{P}_{net,i}$

Elastic Collision: $v_{1i} - v_{2i} = v_{2f} - v_{1f}$

Uniform circular motion

$$|\vec{a}| = \frac{v^2}{r}$$

Newton's Universal Law of Gravitation

$$|F_g| = \frac{G\, m_1 m_2}{r_{12}^2}$$

G=6.67x10^{-11} Nm^2/kg^2

$$g = \frac{G\, M_{Earth}}{R_{Earth}^2}$$

$M_{earth} = 5.97 \times 10^{24}$ kg
$R_{earth} \approx 6400$ km.

Work-Energy Theorem: $W_{net} = K_f - K_i$

$$K = \frac{1}{2} m v^2$$

In 1D $W_F = F\Delta x$

In 2D, $W_F = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta_{F,d}$

$$\equiv \vec{F}\cdot\vec{d} = F_x d_x + F_y d_y$$

$U_g = mgh$ $W_g = -\Delta U_g$ g = 9.8 m/s^2

Spring: $F_s = -kx$ $\Delta U_s = \frac{1}{2} k (x_f^2 - x_i^2)$

$$W_{spring} = -\frac{1}{2} k (x_f^2 - x_i^2) = \frac{1}{2} k (x_i^2 - x_f^2)$$

$$\Delta E_{mech} = \Delta K + \Delta U$$

If the work done by non-conservative forces is zero, the mechanical energy is conserved.

$$E_{mech,f} = E_{mech,i}$$

If the work done by non-conservative forces is not zero, the mechanical energy changes by the amount of the work done by the non-conservative forces.

$$\Delta E_{mech} = E_{mech,f} - E_{mech,i} = W_{non-conservative}$$

Average power $P_{avg} = \frac{W}{\Delta t}$ 1 hp = 746 W

Instantaneous power $P = \left|\vec{F}\right|\left|\vec{v}\right| \cos\theta_{F,v}$

Rotation Counter clockwise: +, Clockwise : -

2π radians = 360 degree

If N-number of revolutions, then θ = 2πN.

Angular displacement $\Delta\theta \equiv \theta_{final} - \theta_{inital}$

Angular velocity $\omega_{ave} \equiv \frac{\Delta\theta}{\Delta t}$ Angular acceleration $\alpha_{ave} \equiv \frac{\Delta\omega}{\Delta t}$

T: period f: frequency

$\omega = 2\pi f = 2\pi / T$ $f = \omega / 2\pi$

$\Delta s = r\Delta\theta$ $v_T = r\omega$ $a_T = r\alpha$ $a_r = \frac{v_T^2}{r} = r\omega^2$

Rotation with constant angular acceleration α

$$\omega_f(t) = \omega_0 + \alpha t \qquad \theta_f(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

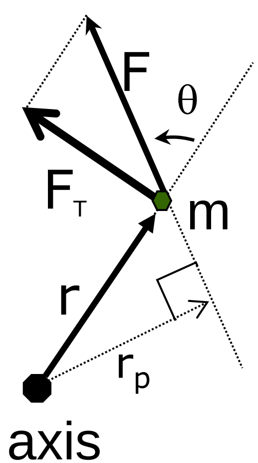
$$\omega_f^2(t) = \omega_0^2 + 2\alpha[\theta_f - \theta_0]$$

Moment of inertia of particles $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots$

Torque

$$|\tau| = rF_T = rF\sin\theta = r_p F$$

$$\tau_{net} = I\alpha$$



Rotation around a fixed axis

$$K = \frac{1}{2} I\omega^2$$

$$U_{gravity} = Mgh_{com}$$

Moment of inertia of uniform rigid bodies