

Formulas PHYS102, Final Exam, Spring 2023

Formulas for Motion in 1-dimension

Displacement = $\Delta x = x_2 - x_1$

Average velocity $v_{avg} = \frac{x_2 - x_1}{t_2 - t_1}$

(Average speed) = $s_{avg} = \frac{\text{(Total distance)}}{\text{(Time change)}}$

Average acceleration $a_{avg} = \frac{v_2 - v_1}{t_2 - t_1}$

Motion with a constant velocity, v $x = x_0 + v t$

Motion with a constant acceleration "a"

$v = v_0 + a t$ $x = x_0 + v_0 t + \frac{1}{2} a t^2$

$v^2 - v_0^2 = 2a(x - x_0)$ $x - x_0 = \frac{1}{2} (v_0 + v) t$

Free fall motion:

Motion with a constant acceleration "-g",
where $g = 9.8 \text{ m/s}^2$ (+y direction points up)

$v = v_0 - g t$ $y = y_0 + v_0 t - \frac{1}{2} g t^2$

$v^2 - v_0^2 = -2g(y - y_0)$

Vector Components vs. magnitude and direction

$$\begin{cases} A_x = A \cos(\theta) & (\text{adjacent}) = (\text{hypotenuse}) \times \cos \theta \\ A_y = A \sin(\theta) & (\text{opposite}) = (\text{hypotenuse}) \times \sin \theta \end{cases}$$

$$\begin{cases} |\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2} \\ \theta = \tan^{-1} \left(\frac{A_y}{A_x} \right) = \arctan \left(\frac{A_y}{A_x} \right) \\ \text{(add/subtract 180 deg if necessary)} \end{cases}$$

Equations for Projectile Motion

If the initial velocity $\vec{v}_0 = (v_{0x}, v_{0y})$

$v_{0x} = v_0 \cos \theta_0$

and the initial position $\vec{r}_0 = (x_0, y_0)$ are given, $v_{0y} = v_0 \sin \theta_0$

X-component (horizontal) motion

Y-component (vertical) motion
(+y direction points up)

$v_x = v_{0x}$

$v_y = v_{0y} - g t$

$x = x_0 + v_{0x} t$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

Range: $(v_0^2/g) * \sin(2\theta)$

$v_y^2 - v_{0y}^2 = -2g(y - y_0)$

Units:

$1 \text{ cm} = 10^{-2} \text{ m}$; $1 \text{ km} = 1000 \text{ m}$

$1 \text{ g} = 10^{-3} \text{ kg}$

$1 \text{ lt} = 1000 \text{ ml}$; $1000 \text{ lt} = 1 \text{ m}^3$

$\Delta y = \left(\frac{v_{0y} + v_y}{2} \right) \cdot t$

Newton's 2nd law: $\vec{F}_{net} = m\vec{a}$

Static friction: $|\vec{f}_S| < |\vec{f}_S^{Max}| = \mu_S |\vec{F}_N|$

Kinetic friction: $|\vec{f}_k| = \mu_k |\vec{F}_N|$

\vec{F}_N = (normal force)

Impulse, momentum

Impulse $\vec{I} = \vec{F}\Delta t$ Momentum $\vec{p} = m\vec{v}$

Impulse-momentum theorem $\vec{I}_{net} = \vec{p}_f - \vec{p}_i$

Conservation of momentum for system

If $F_{net}=0$, then $\vec{P}_{net,f} = \vec{P}_{net,i}$

Elastic Collision: $v_{1i} - v_{2i} = v_{2f} - v_{1f}$

Uniform circular motion

$$|\vec{a}| = \frac{v^2}{r}$$

Newton's Universal Law of Gravitation

$$|F_g| = \frac{G m_1 m_2}{r_{12}^2} \quad G=6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$g = \frac{G M_{Earth}}{R_{Earth}^2}$$

Work-Energy Theorem: $W_{net} = K_f - K_i$

$$K = \frac{1}{2}mv^2$$

In 1D $W_F = F\Delta x$

In 2D, $W_F = |\vec{F}| |\vec{d}| \cos \theta_{F,d}$

$$\equiv \vec{F} \cdot \vec{d} = F_x d_x + F_y d_y$$

$U_g = mgh$ $W_g = -\Delta U_g$ $g = 9.8 \text{ m/s}^2$

Spring: $F_s = -kx$ $\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2)$

W_{spring} = $-\frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2}k(x_i^2 - x_f^2)$

$$\Delta E_{mech} = \Delta K + \Delta U$$

If the work done by non-conservative forces is zero, the mechanical energy is conserved.

$$E_{mech,f} = E_{mech,i}$$

If the work done by non-conservative forces is not zero, the mechanical energy changes by the amount of the work done by the non-conservative forces.

$$\Delta E_{mech} = E_{mech,f} - E_{mech,i} = W_{non-conservative}$$

Average power $P_{avg} = \frac{W}{\Delta t}$ 1 hp = 746 W

Instantaneous power $P = |\vec{F}| |\vec{v}| \cos \theta_{F,v}$

Rotation Counter clockwise: +, Clockwise : -

2π radians = 360 degree

If N-number of revolutions, then $\theta = 2\pi N$.

Angular displacement $\Delta\theta \equiv \theta_{final} - \theta_{initial}$

Angular velocity $\omega_{ave} \equiv \frac{\Delta\theta}{\Delta t}$ Angular acceleration $\alpha_{ave} \equiv \frac{\Delta\omega}{\Delta t}$

T: period f: frequency

$$\omega = 2\pi f = 2\pi/T \quad f = \omega/2\pi$$

$$\Delta s = r\Delta\theta \quad v_T = r\omega \quad a_T = r\alpha \quad a_r = \frac{v_T^2}{r} = r\omega^2$$

Rotation with constant angular acceleration α

$$\omega_f(t) = \omega_0 + \alpha t \quad \theta_f(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega_f^2(t) = \omega_0^2 + 2\alpha[\theta_f - \theta_0]$$

Moment of inertia of particles $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$

Torque

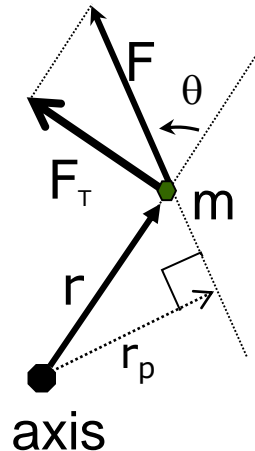
$$|\tau| = r F_T = r F \sin \theta = r_p F$$

$$\tau_{net} = I\alpha$$

Rotation around a fixed axis

$$K = \frac{1}{2} I \omega^2$$

$$U_{gravity} = Mgh_{com}$$

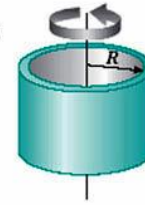


Moment of inertia of uniform rigid bodies

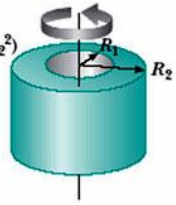
TABLE 10.2

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

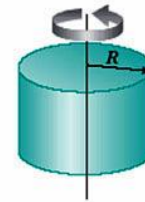
Hoop or thin cylindrical shell
 $I_{CM} = MR^2$



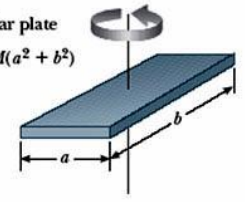
Hollow cylinder
 $I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$



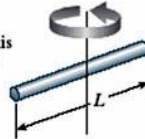
Solid cylinder or disk
 $I_{CM} = \frac{1}{2} MR^2$



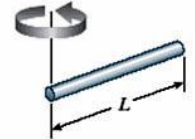
Rectangular plate
 $I_{CM} = \frac{1}{12} M(a^2 + b^2)$



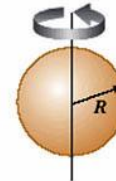
Long, thin rod with rotation axis through center
 $I_{CM} = \frac{1}{12} ML^2$



Long, thin rod with rotation axis through end
 $I = \frac{1}{3} ML^2$



Solid sphere
 $I_{CM} = \frac{2}{5} MR^2$



Thin spherical shell
 $I_{CM} = \frac{2}{3} MR^2$

