Units: SI system: kg (kilo gram), m (meter), s (second), C (coulomb); \( \mathbf{A} \) (ampere)=C/s, V (volt)=J/C, F (farad)=C/V, \( \Omega \) (ohm)=V/A. 1 mm = 10^{-3} m, 1 cm = 10^{-2} m, 1 km = 10^{3} m; 1 N (newton)=kg·m/s², 1 J (joule)=N·m = kg·m²/s², 1 W (watt)=J/s; prefixes: m (milli) 10^{-3}, \( \mu \) (micro) 10^{-6}, n (nano) 10^{-9}, p (pico) 10^{-12}, k (kilo) 10^{3}, M (mega) 10^{6}.

Constants: \( g = 9.8 \text{ m/s}^2, \ k_e = 9 \times 10^9 \text{ N m}^2/\text{C}^2 = 1/(4\pi\varepsilon_0), \ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2), \ \mu_0 = 4\pi \times 10^{-7} \text{ T}⋅\text{m}/\text{A} \). \( e = -1.6 \times 10^{-19} \text{ C}, \ m_e = 9.11 \times 10^{-31} \text{ kg}, \ m_p \approx m_n = 1.67 \times 10^{-27} \text{ kg} \)

Volumes. Cylinder: \( \pi R^2 h \), sphere: \( \frac{4}{3}\pi R^3 \), cone: \( \frac{1}{3}\pi R^2 h \). Areas: Sphere: \( 4\pi R^2 \), circle: \( \pi R^2 \).

Quadratic equations. \( ax^2 + bx + c = 0, x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a) \)

Derivatives/integrals. \( \frac{d}{dx}x^n = nx^{n-1}, \frac{d}{dx}\sin x = \cos x, \frac{d}{dx}\cos x = -\sin x, \frac{d}{dx}e^{-ax} = -ae^{-ax}; \int x^n dx = \frac{1}{n+1}x^{n+1}, \int dx/x = \ln x; \int dx/\sqrt{a^2 + x^2} = \ln \left(x + \sqrt{a^2 + x^2}\right); \int dx/(a^2 + x^2)^{\frac{3}{2}} = x/\left(a^2\sqrt{a^2 + x^2}\right); \int x dx/(a^2 + x^2)^{\frac{5}{2}} = -1/\left(\sqrt{a^2 + x^2}\right); \int \sin x dx = -\cos x; \int \cos x dx = \sin x \).

Vectors. If \( \vec{c} = \vec{a} + \vec{b} \), then \( c_x = a_x + b_x, \ c_y = a_y + b_y, \ c_z = a_z + b_z \) and \( c = \sqrt{c_x^2 + c_y^2 + c_z^2} \). Dot product: \( \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = a \cdot b \cdot \cos \theta \). Cross product: \( i \times j = k, \ j \times k = i, \ k \times i = j \). \( \vec{A} \times \vec{B} = (A_y B_z - A_z B_y)i + (A_z B_x - A_x B_z)j + (A_x B_y - A_y B_x)k \); \( \vec{A} \times \vec{B} = A \cdot B \cdot \sin \alpha \).

Coulombs Law: \( F = k_e \frac{q_1 q_2}{r^2} \), r-distance between charges; in vector form \( \vec{F} = k_e \frac{q_1 q_2 \hat{r}}{r^2} \), \( \hat{r} = \vec{r}/r \) - unit vector from charge \( q_1 \) to \( q_2 \), \( k_e = 9 \times 10^9 \ldots \) Superposition: if charge \( q_1 \) acts on \( q_0 \) with \( \vec{F}_{01} \), charge \( q_2 \) acts on \( q_0 \) with \( \vec{F}_{02} \), etc., then \( \vec{F}_{\text{net}} = \vec{F}_{01} + \vec{F}_{02} + \ldots \)

Electric field. Definition: \( \vec{E} = \vec{F}_0/q_0 \) (charge \( q_0 \) is a ”probe”). Field from a charge \( q \): \( E = k_e \frac{q}{\hat{r}^2} \), r-distance between charge and observation point; in vector form \( \vec{E} = k_e \frac{q}{r^2} \hat{r} \), \( \hat{r} = \vec{r}/r \) - unit vector from charge \( q \) to the observation point, \( k_e = 9 \times 10^9 \ldots \) Superposition: consider charges \( q_1, q_2 \), etc. and the observation point. If \( q_1 \) creates field \( \vec{E}_1 \) at the observation point, \( q_2 \) creates field \( \vec{E}_2 \), etc., then \( \vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots \)

Force on a charge placed in external field \( \vec{E} \): \( \vec{F} = q\vec{E} \).

Gauss Law. Flux through a small area \( A \) with \( \vec{A} \) along the normal to the surface: \( \Phi = \vec{E} \cdot \vec{A} \). Flux through a closed surface: \( \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0 \). Field \( E(r) \) from a uniformly charged spherical shell with radius \( R \) and charge \( Q \): \( E(r < R) = 0, E(r > R) = k_e Q/r^2 \). Field \( E(r) \) from a uniformly charged infinite line with linear charge density \( \lambda \): \( E(r) = \lambda/(2\pi \varepsilon_0 r) = 2k_e \lambda /r \). Field \( E \) from a uniformly charged infinite non-conducting plane with surface charge density \( \sigma \): \( E = \sigma/(2\varepsilon_0) = 2\pi k_e \sigma \).

Potential. Definition: \( V = U/q_0 \). Work: \( W_{AB} = U_A - U_B = -q \Delta V \). Two charges: \( U = k_e q_1 q_2 /r \). Point charge: \( V(r) = k_e q/r \). Superposition: \( V(\vec{r}) = k_e (q_1/r_1 + q_2/r_2 + \ldots) \) (with \( r_1 - \) distance from \( q_1 \) to observation point, etc.). Potential from field: \( V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}; \) for uniform field: \( \Delta V = -Ed \). Field from potential: \( E_x = -dV/dx, \ldots \) Conducting sphere with charge \( Q \) and radius \( R \): \( V(r) = k_e Q/r, r \geq R \) and \( V(r) = k_e Q/R, r \leq R \).

Conductors. Inside the body of a conductor: \( \vec{E} = 0, V =\text{const} \), no charge. Extra charge - goes to outer surface. Field near the surface of a conductor, outside, is \( E = \sigma/\varepsilon_0 \); potential - continuous. Inner surfaces (surfaces of cavities) - uncharged, unless an extra charge is placed inside a cavity.
Capacitors. Definition: $C = Q/V$. Single sphere: $C = 4\pi \varepsilon_0 R$. Spherical: $C = 4\pi \varepsilon_0 R_1 R_2/(R_2 - R_1)$. Parallel plate capacitor: $C = \varepsilon_0 \kappa / d$. Energy: $U_C = Q^2/(2C) = V^2 C/2$. Connections: parallel (same voltage) $C_{eq} = C_1 + C_2 + \ldots$; series (same charge) $1/C_{eq} = 1/C_1 + 1/C_2 + \ldots$ or $C_{eq} = C_1 C_2/(C_1 + C_2)$ (for two only). $Q_{tot} = V C_{eq}$.

Current. Definitions: current $I = \Delta q/\Delta t \simeq dq/dt$, density of current $J = I/A$, with $A$ - cross-sectional area. Ohm’s law: $I = V/R$ with $R$-resistance. For wire of length $L$: $R = \rho L / A$, with $\rho$ - resistivity of material. $J = E / \rho$. Power: $P = IV = I^2 R = V^2 / R$. Simple connections: series (same current) $R_{eq} = R_1 + R_2 + \ldots$; parallel (same voltage) $1/R_{eq} = 1/R_1 + 1/R_2 + \ldots$ or $R_{eq} = R_1 R_2/(R_1 + R_2)$ (for two only). Microscopic picture: $J = \varepsilon_{0u}$.

Multiloop circuits and Kirchhoff’s equations. Potential changes: $+\mathcal{E}$ when crossing battery from negative to positive terminal; $-IR$ when traveling through a resistor along the current. Loop equation: sum of potential drop around a close loop is zero. Junction equation: sum of incoming currents equals sum of outgoing currents.

Charging a capacitor: $q(t) = \mathcal{E} C \left( 1 - e^{-t/\tau} \right)$ with $\tau = RC$; $V_C(t) = \frac{q}{C} = \mathcal{E} \left( 1 - e^{-t/\tau} \right)$, $i(t) = i_{max} e^{-t/\tau}$ with $i_{max} = \frac{\mathcal{E}}{R}$. Discharging a capacitor: $\tau = RC$, $q(t) = Q_0 e^{-t/\tau}$, $V(t) = V_0 e^{-t/\tau}$ with $V_0 = \frac{Q_0}{C}$, $i(t) = i_0 e^{-t/\tau}$ with $i_0 = \frac{Q_0}{RC}$.

Magnetic force. Force on a particle: $\vec{F}_m = q \vec{v} \times \vec{B}$. Revolution in magnetic field: radius $r = mv/(qB)$, period $T = 2\pi m/(qB)$. Force on a wire: $\vec{F}_w = I \vec{L} \times \vec{B}$. Interaction between two parallel wires: $F = \mu_0 I_1 I_2 / (2\pi d)$ with $\mu_0 = 4\pi \times 10^{-7}$. Magnetic moment for a coil with $N$ turns: $\vec{\mu} = N I \vec{A}$; potential energy: $U = -\mu \cdot \vec{B}$; torque $\vec{\tau} = \mu \times \vec{B}$.

Fields from currents. Straight wire: $B = \mu_0 I / (2\pi d)$. Bio-Savart: $d\vec{B} = (\mu_0 / 4\pi) I d\vec{s} \times \vec{r} / r^3$. At the center of ring current of radius $R$: $B_{ring} = \mu_0 I / (2R)$; for a circular arc with angle $\theta$ (in radians): $B = B_{ring} \times \theta / (2\pi)$. Amper’s Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$. Field in a solenoid: $B = \mu_0 I n$, with $n$ - density of turns.

Electromagnetic induction (Faraday Law). EMF $\mathcal{E} = -d\Phi_B / dt$. $\Phi_B \simeq \vec{A} \cdot \vec{B}$. If area $A$ = const, $\mathcal{E} = -A dB / dt$; if $B$ = const, $\mathcal{E} = -B dA / dt$; moving rod: $\mathcal{E} = lvB$. Induced current: $I_{ind} = \mathcal{E} / R$. Direction (Lenz rule): the flux due to induced current tries to oppose the changes in the original flux. Self-induction. Magnetic flux through a conductor with current $I$: $\Phi = LI$, $L$-inductance. For a solenoid with length $l$, cross-sectional area $A$ and density of turns $n$, one has $L = \mu_0 An$. Self-induced EMF: $\mathcal{E}_L = -L dI / dt$. Energy: $U_L = \frac{1}{2} LI^2$.

RL circuits. Time constant $\tau_L = L / R$. Decay of current $I(t) = I_0 \exp(-t / \tau_L)$. Build up of current $I(t) = \mathcal{E} / R \ast [1 - \exp(-t / \tau_L)]$. At $t = 0$ inductor acts as infinite resistance; at $t \to \infty$ acts as a piece of wire.

LC and driven LRC circuits. Free oscillations in LC circuit: natural (or resonant) frequency $\omega_0 = 1 / \sqrt{LC}$ (in rad/s) or $\omega_0 = \omega_0 / (2\pi)$ (in Hz). Energy conservation: $\frac{1}{2} Q^2 / C + \frac{1}{2} L I^2 = \text{const}$ or $\frac{1}{2} Q_{max}^2 / C = \frac{1}{2} L_{max}^2$. Driven: $f_d$ (in Hz) - driving frequency; $\omega_d = 2\pi f_d$. Inductive reactance: $X_L = \omega_d L$; capacitive reactance: $X_C = 1 / (\omega_d C)$. Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Amplitude of current: $I_m = \mathcal{E}_m / Z$. Phase angle: $\tan \phi = (X_L - X_C) / R$. Resonance: $\omega_d = \omega_0$, $X_L = X_C$, $Z = R = \text{min}$, $I_m = \mathcal{E}_m / R = \text{max}$. RMS values: $I_{RMS} = I_m / \sqrt{2}$, $V_{RMS} = \mathcal{E}_m / \sqrt{2}$; $I_{RMS} = V_{RMS} / Z$. Power $P = I_{RMS}^2 R$. Transformers: $V_2 = V_1 \ast N_2 / N_1$, $I_2 = I_1 \ast N_1 / N_2$. 

2
From Phys 111:

**Kinematics:** $v = dx/dt$, $a = dv/dt = d^2x/dt^2$. Circular motion with constant speed: $\omega = v/R$, $a_c = v^2/R = \omega^2 R$, towards center.

**The three Laws of motion:** (1) If $\vec{F}_{net} = 0$ then $\vec{v} = const$; (2) $\vec{F}_{net} = m\vec{a}$; (3) $\vec{F}_{21} = -\vec{F}_{12}$

**Work and power.** $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$. Power: $P = W/\Delta t = \vec{F} \cdot \vec{v}$.

**Kinetic energy and work-energy theorem:** $K = \frac{1}{2}mv^2$, $\Delta K = W$ where $W$ is the net work (i.e. work by all forces).

**Potential energy.** For conservative forces (with path-independent work) introduce $U(\vec{r})$ so that $W_{AB} = U_A - U_B = -\Delta U$. If only conservative forces, then energy conservation: $K + U = const.$