

Units: SI system: kg (kilogram), m (meter), s (second), C (coulomb); \mathcal{A} (ampere)=C/s, V (volt)=J/C, F (farad)=C/V, Ω (ohm)=V/ \mathcal{A} . 1 mm = 10^{-3} m, 1 cm = 10^{-2} m, 1 km = 10^3 m; 1 N (newton)=kg·m/s², 1 J (joule)=N·m = kg·m²/s², 1 W (watt)=J/s; prefixes: m (milli) 10^{-3} , μ (micro) 10^{-6} , n (nano) 10^{-9} , p(pico) 10^{-12} , k (kilo) 10^3 , M (mega) 10^6 .

Constants: $g = 9.8 \text{ m/s}^2$, $k_e = 9 * 10^9 \text{ N m}^2/\text{C}^2 = 1/(4\pi\epsilon_0)$, $\epsilon_0 = 8.85 * 10^{-12} \text{ C}^2/(\text{N m}^2)$, $\mu_0 = 4\pi * 10^{-7} \text{ T-m/A}$. $e = -1.6 * 10^{-19} \text{ C}$, $m_e = 9.11 * 10^{-31} \text{ kg}$, $m_p \simeq m_n = 1.67 * 10^{-27} \text{ kg}$

Volumes. Cylinder: $\pi R^2 h$, sphere: $\frac{4}{3}\pi R^3$, cone: $\frac{1}{3}\pi R^2 h$. **Areas:** Sphere: $4\pi R^2$, circle: πR^2 .

Quadratic equation. $ax^2 + bx + c = 0$, $x = \left(-b \pm \sqrt{b^2 - 4ac}\right) / (2a)$

Derivatives/integrals. $\frac{d}{dx}x^n = nx^{n-1}$, $\frac{d}{dx}\sin x = \cos x$, $\frac{d}{dx}\cos x = -\sin x$, $\frac{d}{dx}e^{-ax} = -ae^{-ax}$; $\int x^n dx = \frac{1}{n+1}x^{n+1}$, $\int dx/x = \ln x$; $\int dx / \sqrt{a^2 + x^2} = \ln\left(x + \sqrt{a^2 + x^2}\right)$; $\int dx (a^2 + x^2)^{-\frac{3}{2}} = x / (a^2\sqrt{a^2 + x^2})$; $\int x dx (a^2 + x^2)^{-\frac{3}{2}} = -1 / (\sqrt{a^2 + x^2})$; $\int \sin x dx = -\cos x$; $\int \cos x dx = \sin x$.

Vectors. If $\vec{c} = \vec{a} + \vec{b}$, then $c_x = a_x + b_x$, $c_y = a_y + b_y$, $c_z = a_z + b_z$ and $c = \sqrt{c_x^2 + c_y^2 + c_z^2}$. Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = a * b * \cos \theta$. Cross product: $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$; $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$; $|\vec{A} \times \vec{B}| = A * B * \sin \alpha$.

Coulombs Law: $F = k_e \frac{q_1 q_2}{r^2}$, r -distance between charges; in vector form $\vec{F} = k_e \frac{q_1 q_2}{r^2} \hat{r}$, $\hat{r} = \vec{r}/r$ - unit vector from charge q_1 to q_2 , $k_e = 9 * 10^9 \dots$ Superposition: if charge q_1 acts on q_0 with \vec{F}_{01} , charge q_2 acts on q_0 with \vec{F}_{02} , etc., then $\vec{F}_{\text{net on } q_0} = \vec{F}_{01} + \vec{F}_{02} + \dots$

Electric field. Definition: $\vec{E} = \vec{F}_0/q_0$ (charge q_0 is a "probe"). Field from a charge q : $E = k_e \frac{q}{r^2}$, r -distance between charge and observation point; in vector form $\vec{E} = k_e \frac{q}{r^2} \hat{r}$, $\hat{r} = \vec{r}/r$ - unit vector from charge q to the observation point, $k_e = 9 * 10^9 \dots$ Superposition: consider charges q_1, q_2 , etc. and the observation point. If q_1 creates field \vec{E}_1 at the observation point, q_2 creates field \vec{E}_2 , etc., then $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$
Force on a charge placed in external field \vec{E} : $\vec{F} = q\vec{E}$.

Gauss Law. Flux through a small area A with \vec{A} along the normal to the surface: $\Phi = \vec{E} \cdot \vec{A}$. Flux through a closed surface: $\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\epsilon_0$. Field $E(r)$ from a uniformly charged spherical shell with radius R and charge Q : $E(r < R) = 0$, $E(r > R) = k_e Q/r^2$. Field $E(r)$ from a uniformly charged infinite line with linear charge density λ : $E(r) = \lambda/(2\pi\epsilon_0 r) = 2k_e \lambda/r$. Field E from a uniformly charged infinite non-conducting plane with surface charge density σ : $E = \sigma/(2\epsilon_0) = 2\pi k_e \sigma$.

Potential. Definition: $V = U/q_0$. Work: $W_{AB} = U_A - U_B = -q\Delta V$. Two charges: $U = k_e q_1 q_2 / r$. Point charge: $V(r) = k_e q / r$. Superposition: $V(\vec{r}) = k_e (q_1/r_1 + q_2/r_2 + \dots)$ (with r_1 - distance from q_1 to observation point, etc.). Potential from field: $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$; for uniform field: $\Delta V = -Ed$. Field from potential: $E_x = -dV/dx, \dots$ Conducting sphere with charge Q and radius R : $V(r) = k_e Q/r$, $r \geq R$ and $V(r) = k_e Q/R$, $r \leq R$.

Conductors. Inside the body of a conductor: $\vec{E} = 0$, $V = \text{const}$, no charge. Extra charge - goes to outer surface. Field near the surface of a conductor, *outside*, is $E = \sigma/\epsilon_0$; potential - continuous. Inner surfaces (surfaces of cavities) - uncharged, unless an extra charge is placed inside a cavity.

Capacitors. Definition: $C = Q/V$. Single sphere: $C = 4\pi\epsilon_0 R$. Spherical: $C = 4\pi\epsilon_0 R_1 R_2 / (R_2 - R_1)$. Parallel plate capacitor: $C = A\epsilon_0\kappa/d$. Energy: $U_C = Q^2 / (2C) = V^2 C / 2$. Connections: parallel (same voltage) $C_{eq} = C_1 + C_2 + \dots$; series (same charge) $1/C_{eq} = 1/C_1 + 1/C_2 + \dots$ or $C_{eq} = C_1 C_2 / (C_1 + C_2)$ (for two only). $Q_{tot} = V C_{eq}$.

Current. Definitions: current $I = \Delta q / \Delta t \simeq dq/dt$, density of current $J = I/A$, with A - cross-sectional area. Ohm's law: $I = V/R$ with R -resistance. For wire of length L : $R = \rho L/A$, with ρ - resistivity of material. $J = E/\rho$. Power: $P = IV = I^2 R = V^2/R$. Simple connections: series (same current) $R_{eq} = R_1 + R_2 + \dots$; parallel (same voltage) $1/R_{eq} = 1/R_1 + 1/R_2 + \dots$ or $R_{eq} = R_1 R_2 / (R_1 + R_2)$ (for two only). Microscopic picture: $J = env_d$.

Multiloop circuits and Kirchoff's equations. Potential changes: $+\mathcal{E}$ when crossing battery from negative to positive terminal; $-IR$ when traveling through a resistor along the current. Loop equation: potential drop around a close loop is zero. Junction equation: sum of incoming currents equals sum of outgoing currents.

Charging a capacitor: $q(t) = \mathcal{E}C(1 - e^{-t/\tau})$ with $\tau = RC$; $V_C(t) = \frac{q}{C} = \mathcal{E}(1 - e^{-t/\tau})$, $i(t) = i_{max}e^{-t/\tau}$ with $i_{max} = \frac{\mathcal{E}}{R}$. Discharging a capacitor: $\tau = RC$, $q(t) = Q_0 e^{-t/\tau}$, $V(t) = V_0 e^{-t/\tau}$ with $V_0 = \frac{Q_0}{C}$; $i(t) = i_0 e^{-t/\tau}$ with $i_0 = \frac{Q_0}{RC}$.

Magnetic force. Force on a particle: $\vec{F}_m = q\vec{v} \times \vec{B}$. Revolution in magnetic field: radius $r = mv/(qB)$, period $T = 2\pi m/(qB)$. Force on a wire: $\vec{F}_w = I\vec{L} \times \vec{B}$. Interaction between two parallel wires: $F = \mu_0 I_1 I_2 L / (2\pi d)$ with $\mu_0 = 4\pi * 10^{-7}$. Magnetic moment for a coil with N turns: $\vec{\mu} = NI\vec{A}$; potential energy: $U = -\vec{\mu} \cdot \vec{B}$; torque $\vec{\tau} = \vec{\mu} \times \vec{B}$.

Fields from currents. Straight wire: $B = \mu_0 I / (2\pi d)$. Bio-Savart: $d\vec{B} = (\mu_0 / 4\pi) Id\vec{s} \times \vec{r} / r^3$. At the center of ring current of radius R : $B_{ring} = \mu_0 I / (2R)$; for a circular arc with angle θ (in radians): $B = B_{ring} \times \theta / (2\pi)$. Amper's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$. Field in a solenoid: $B = \mu_0 I n$, with n - density of turns.

Electromagnetic induction (Faraday Law). EMF $\mathcal{E} = -d\Phi_B / dt$. $\Phi_B \simeq \vec{A} \cdot \vec{B}$. If area $A = \text{const}$, $\mathcal{E} = -A dB/dt$; if $B = \text{const}$, $\mathcal{E} = -B dA/dt$; moving rod: $\mathcal{E} = lvB$. Induced current: $I_{ind} = \mathcal{E}/R$. Direction (Lenz rule): the flux due to induced current tries to oppose the changes in the original flux. **Self-induction.** Magnetic flux through a conductor with current I : $\Phi = LI$, L -inductance. For a solenoid with length l , cross-sectional area A and density of turns n , one has $L/l = \mu_0 A n^2$. Self-induced EMF: $\mathcal{E}_L = -L dI/dt$. Energy: $U_L = \frac{1}{2} LI^2$.
$$\vec{\mathcal{E}} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

RL circuits. Time constant $\tau_L = L/R$. Decay of current $I(t) = I_0 \exp(-t/\tau_L)$. Build up of current $I(t) = \mathcal{E}/R * [1 - \exp(-t/\tau_L)]$. At $t = 0$ inductor acts as infinite resistance; at $t \rightarrow \infty$ acts as a piece of wire.

LC and driven LRC circuits. Free oscillations in LC circuit: natural (or resonant) frequency $\omega_0 = 1/\sqrt{LC}$ (in rad/s) or $f_0 = \omega_0 / (2\pi)$ (in Hz). Energy conservation: $\frac{1}{2} q^2 / C + \frac{1}{2} L i^2 = \text{const}$ or $\frac{1}{2} Q_{max}^2 / C = \frac{1}{2} L I_{max}^2$. **Driven:** f_d (in Hz) - driving frequency; $\omega_d = 2\pi f_d$. Inductive reactance: $X_L = \omega_d L$; capacitive reactance: $X_C = 1/(\omega_d C)$. Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$. Amplitude of current: $I_m = \mathcal{E}_m / Z$. Phase angle: $\tan \phi = (X_L - X_C) / R$. Resonance: $\omega_d = \omega_0$, $X_L = X_C$, $Z = R = \text{min}$, $I_m = \mathcal{E}_m / R = \text{max}$. RMS values: $I_{RMS} = I_m / \sqrt{2}$, $V_{RMS} = \mathcal{E}_m / \sqrt{2}$; $I_{RMS} = V_{RMS} / Z$. Power $P = I_{RMS}^2 R$. Transformers: $V_2 = V_1 * N_2 / N_1$, $I_2 = I_1 * N_1 / N_2$.

From Phys 111:

Kinematics: $v = dx/dt$, $a = dv/dt = d^2x/dt^2$. Circular motion with constant speed: $\omega = v/R$, $a_c = v^2/R = \omega^2 R$, towards center.

The three Laws of motion: (1) If $\vec{F}_{net} = 0$ then $\vec{v} = const$; (2) $\vec{F}_{net} = m\vec{a}$; (3) $\vec{F}_{21} = -\vec{F}_{12}$

Work and power. $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$. Power: $P = W/\Delta t = \vec{F} \cdot \vec{v}$.

Kinetic energy and work-energy theorem: $K = \frac{1}{2}mv^2$, $\Delta K = W$ where W is the *net* work (i.e. work by all forces).

Potential energy. For conservative forces (with path-independent work) introduce $U(\vec{r})$ so that $W_{AB} = U_A - U_B = -\Delta U$. If *only* conservative forces, then energy conservation: $K + U = const$.