Formulas for Motion in 1-dimension

Displacement = \( \Delta x = x_2 - x_1 \)

Average velocity \( v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} \)

(Average speed ) = \( s_{avg} = \frac{(Total \ distance)}{(Time \ change)} \)

Average acceleration \( a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} \)

Motion with a constant velocity, \( v \quad x = x_0 + v \ t \)

Motion with a constant acceleration “a”
\( v = v_0 + a \ t \quad x = x_0 + v_0 t + \frac{1}{2} at^2 \)
\( v^2 - v_0^2 = 2a(x-x_0) \)

Free fall motion:
Motion with a constant acceleration “-g”, where \( g = 9.8 \ m/s^2 \) (+y direction points up)
\( v = v_0 - g \ t \quad y = y_0 + v_0 t - \frac{1}{2} gt^2 \)
\( v^2 - v_0^2 = -2g(y-y_0) \)

Vector Components vs. magnitude and direction

\[
\begin{array}{l}
A_x = A \cos(\theta) \quad (\text{adjacent}) = (\text{hypotenuse}) \times \cos \theta \\
A_y = A \sin(\theta) \quad (\text{opposite}) = (\text{hypotenuse}) \times \sin \theta \\
|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2} \\
\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \arctan\left(\frac{A_y}{A_x}\right) \\
\end{array}
\]

(add/subtract 180 deg if necessary)

Equations for Projectile Motion

If the initial velocity \( \vec{v}_0 = (v_{0x}, v_{0y}) \)
and the initial position \( \vec{r}_0 = (x_0, y_0) \) are given.

X-component (horizontal) motion
\( v_x = v_{0x} \)
\( x = x_0 + v_{0x} t \)

Y-component (vertical) motion (+y direction points up)
\( v_y = v_{0y} - gt \)
\( y = y_0 + v_{0y} t - \frac{1}{2} gt^2 \)
\( v_y^2 - v_{0y}^2 = -2g(y-y_0) \)
Newton's 2nd law: \( \vec{F}_{net} = m\vec{a} \)

Static friction: \[ |\vec{f}_S| < |\vec{f}_{S,\text{Max}}| = \mu_S |\vec{F}_N| \]

Kinetic friction: \[ |\vec{f}_k| = \mu_k |\vec{F}_N| \]

\( \vec{F}_N \) = (normal force)

**Work-Energy Theorem:** \( W_{\text{net}} = K_f - K_i \)

\( K = \frac{1}{2}mv^2 \)

In 1D \( W_F = F\Delta x \)

In 2D, \( W_F = |\vec{F}| |\vec{d}| \cos \theta_{F,d} \equiv \vec{F} \cdot \vec{d} = F_x d_x + F_y d_y \)

\( U_g = mgh \)

\( W_g = -\Delta U_g \)

\( E_{\text{mech}} \equiv K + U_g \)

\( g = 9.8 \text{ m/s}^2 \)

Average power \( P_{\text{avg}} = \frac{W}{\Delta t} \)

1 hp = 746 W

Instantaneous power \( P = |\vec{F}| |\vec{v}| \cos \theta_{F,v} \)

**Impulse, momentum**

Impulse \( \vec{I} = \vec{F}\Delta t \)

Momentum \( \vec{p} = m\vec{v} \)

Impulse-momentum theorem \( \vec{I}_{\text{net}} = \vec{p}_f - \vec{p}_i \)

Conservation of momentum for system

If \( \vec{I}_{\text{net,ext}} = 0 \), then \( \vec{p}_{\text{net},f} = \vec{p}_{\text{net},i} \)

If the work done by non-conservative forces is zero, the mechanical energy is conserved.

\( E_{\text{mech},f} = E_{\text{mech},i} \)

If the work done by non-conservative forces is not zero, the mechanical energy changes by the amount of the work done by the non-conservative forces.

\( \Delta E_{\text{mech}} = E_{\text{mech},f} - E_{\text{mech},i} = W_{\text{non-conservative}} \)
Rotation

Counter clockwise: +,  Clockwise : -

Moment of inertia of uniform rigid bodies

Rotation

\[
2\pi \text{ radians} = 360 \text{ degree}
\]

Angular displacement \( \Delta \theta \equiv \theta_{\text{final}} - \theta_{\text{initial}} \)

Angular velocity \( \omega_{\text{ave}} \equiv \frac{\Delta \theta}{\Delta t} \)  Angular acceleration \( \alpha_{\text{ave}} \equiv \frac{\Delta \omega}{\Delta t} \)

\[ T: \text{ period} \quad f: \text{ frequency} \]

\[ \omega = 2\pi f = 2\pi/T \quad f = \omega/2\pi \]

\[ \Delta s = r \Delta \theta \quad v_T = r \omega \quad a_T = r \alpha \quad a_r = \frac{v_T^2}{r} = r \omega^2 \]

Rotation with constant angular acceleration \( \alpha \)

\[ \omega_f(t) = \omega_0 + \alpha t \]

\[ \theta_f(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \]

\[ \omega_f^2(t) = \omega_0^2 + 2\alpha[\theta_f - \theta_0] \]

Moment of inertia of particles \( I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots \)

Torque \( |\tau| = r F_T = r F \sin \theta = r_p F \)

Angular momentum \( |L| = I \omega = r p_T = r p \sin \theta = r_p p \)

\[ \tau_{\text{net}} = I \alpha \]

Rotation around a fixed axis \( K = \frac{1}{2} I \omega^2 \)

\[ U_{\text{gravity}} = Mgh_{\text{com}} \]

Net angular momentum,

\[ L_{\text{net}} \text{ is conserved if } r_{\text{net,ext}} = 0 \]

Condition for equilibrium

\[ F_{\text{net},x} = 0 \quad F_{\text{net},y} = 0 \quad \tau_{\text{net}} = 0 \]

**TABLE 10.2**  Moment of inertia of uniform rigid bodies

<table>
<thead>
<tr>
<th>Moments of Inertia of Homogeneous Rigid Objects with Different Geometries</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hoop or thin cylindrical shell</strong></td>
</tr>
<tr>
<td>( I_{CM} = MR^2 )</td>
</tr>
<tr>
<td><strong>Hollow cylinder</strong></td>
</tr>
<tr>
<td>( I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2) )</td>
</tr>
<tr>
<td><strong>Solid cylinder or disk</strong></td>
</tr>
<tr>
<td>( I_{CM} = \frac{1}{2} MR^2 )</td>
</tr>
<tr>
<td><strong>Rectangular plate</strong></td>
</tr>
<tr>
<td>( I_{CM} = \frac{1}{12} M(a^2 + b^2) )</td>
</tr>
<tr>
<td><strong>Long, thin rod with rotation axis through center</strong></td>
</tr>
<tr>
<td>( I_{CM} = \frac{1}{12} ML^2 )</td>
</tr>
<tr>
<td><strong>Long, thin rod with rotation axis through end</strong></td>
</tr>
<tr>
<td>( I = \frac{1}{3} ML^2 )</td>
</tr>
<tr>
<td><strong>Solid sphere</strong></td>
</tr>
<tr>
<td>( I_{CM} = \frac{2}{5} MR^2 )</td>
</tr>
<tr>
<td><strong>Thin spherical shell</strong></td>
</tr>
<tr>
<td>( I_{CM} = \frac{2}{3} MR^2 )</td>
</tr>
</tbody>
</table>