Formulas - exam 2

**Constants and units:**
\[ g = 9.8 \text{ m/s}^2, \quad 1 \text{mm} = 10^{-3} \text{m}, \quad 1 \text{cm} = 10^{-2} \text{m}, \quad 1 \text{km} = 10^3 \text{m}, \quad 1 \text{in} = 2.54 \text{cm}, \quad 1 \text{mi} = 1609 \text{m}; \quad 1 \text{N (newton)} = \text{kg} \cdot \text{m/s}^2, \quad 1 \text{J (joule)} = \text{N} \cdot \text{m} = \text{kg} \cdot \text{m}^2/\text{s}^2, \quad 1 \text{W (watt)} = \text{J/s} \]

**Volumes.** Cylinder: \( \pi R^2 h, \) sphere: \( \frac{4}{3} \pi R^3, \) cone: \( \frac{1}{3} \pi R^2 h \)

**Quadratic equation.** \( ax^2 + bx + c = 0, \ x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/\left(2a\right) \)

**Derivatives/integrals.** \( \frac{d}{dt} t^n = nt^{n-1}, \ \int t^n \, dt = \frac{1}{n+1} t^{n+1} \)

**Vectors.** If \( \vec{c} = \vec{a} + \vec{b} \), then \( c_x = a_x + b_x, \ c_y = a_y + b_y, \ c_z = a_z + b_z \) and \( c = \sqrt{c_x^2 + c_y^2 + c_z^2}. \) Dot product: \( \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab \cos \alpha. \) Cross product: \( \hat{\vec{c}} = \hat{i} \times \hat{j} = \hat{k}, \ \hat{j} \times \hat{k} = \hat{i}, \ \hat{i} \times \hat{i} = 0. \)

**Kinematics.**
1D: \( v = dx/dt, \ a = dv/dt = d^2x/dt^2. \) Constant \( a: \) \( v - v_0 = at, \ x = x_0 + v_0 t + \frac{1}{2} at^2. \) 2D: \( \vec{v} = \vec{v}_0 + \vec{a} t, \ \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \)

Circular motion with constant speed: \( \omega = \omega_0, \) \( \vec{a} = -\omega^2 \vec{r} \) towards center.

The three laws of motion:
1. If \( \vec{F}_{\text{net}} = 0 \) then \( \vec{v} = \text{const}; \)
2. \( \vec{F}_{\text{net}} = m\vec{a}; \)
3. \( \vec{F}_{21} = -\vec{F}_{12} \)

Specific forces. Gravity: \( mg \) (down). Normal \( N \) - perpendicular to surface; tension \( T \) - constant along the string. Spring force: \( \vec{F} = -kx \) (k is spring constant).

Friction - parallel to surface: kinetic: \( f_k = \mu_k N; \) static: \( f_s \leq \mu_s N \) with \( N = mg \) (horizontal plane) or \( N = mg \cos \theta \) (inclined plane).

Inclined plane. Components of gravity: \( mg \sin \theta \) (parallel to plane, downhill) and \( mg \cos \theta \) (perpendicular to plane).

Centripetal motion: \( F_{\text{net}} = mv^2/R; \) direction of \( \vec{F}_{\text{net}} \) - towards center of revolution.

**Work and power.** Constant force \( W = F \cdot \left(\vec{r}_2 - \vec{r}_1\right) = F_X \Delta x + F_Y \Delta y + F_z \Delta z \) (or, \( W = F \Delta r \cos \alpha \)); general: \( W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}. \) Power: \( P = W/\Delta t = \vec{F} \cdot \vec{v}. \) Work by specific forces: gravity: \( W_g = -mg \Delta y \) (and \( \Delta x \) does not matter); normal: \( W_N = 0; \)

Kinetic energy and work-energy theorem: \( K = \frac{1}{2}mv^2, \ \Delta K = W \) where \( W \) is the net work (i.e. work by all forces).

Potential energy. For conservative forces (with path-independent work) introduce \( U (\vec{r}) \) so that \( W_{AB} = U_A - U_B = -\Delta U. \) For specific forces: gravity: \( U_g = mgh; \) spring: \( U_s = \frac{1}{2}kx^2. \)

If only conservative forces, then energy conservation:
\[ K + U = \text{const} \]

If also non-conservative forces (e.g., friction) with work \( W_{\text{non-cons}} \), then \( \Delta (K + U) = W_{\text{non-cons}} \)

**Momentum.** \( \vec{p} = m\vec{v}, \ \vec{P}_{\text{tot}} = \sum m_i \vec{v}_i. \) Impulse \( \Delta \vec{p} = \vec{F}_{\text{ext}} \Delta t. \) Conservation: if \( \vec{F}_{\text{ext}} = 0, \) then \( \vec{P}_{\text{tot}} = \text{const} \) (e.g., in collisions).

**Rotation (kinematics):** If \( N \)-number of revolutions, then \( \theta = 2\pi N. \) Angular velocity \( \omega = d\theta/dt \) (in rad/s); ang. acceleration \( \alpha = d\omega/dt \) (in rad/s²). If \( \alpha = \text{const}, \) then \( \theta = \frac{\omega_0 + \omega t}{2} + \frac{1}{2} \alpha t^2 \) and \( \omega = \omega_0 + \alpha t. \) Connection with linear: \( \omega = v/r, \alpha = a/r \)

**Dynamics:** \( K = \frac{1}{2}I\omega^2; \) moment of inertia. For point masses: \( I = \sum m_i r_i^2 \); for solid bodies \( I = \int dV \rho r^2 \); Specific I's: rod (about center) \( ML^2/12; \) rod (about end) \( ML^2/3; \) hoop \( MR^2; \) disk \( MR^2/2; \) solid sphere \( \frac{2}{5}MR^2. \) Parallel axes theorem: \( I = I_{CM} + MD^2 \)

torque \( \tau = FR \sin \phi = Fd. \) The 2nd Law for rotation: \( \tau = I\alpha. \)

Rotation + linear: \( K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \) If \( \omega = v/R \) (e.g., rolling) \( K = \frac{1}{2}mR^2(1 + I/(MR^2)) \)