Formulas - exam 1

Constants and units: $g = 9.8\, m/s^2$, $1\, mm = 10^{-3}\, m$, $1\, cm = 10^{-2}\, m$, $1\, km = 10^3\, m$, $1\, in = 2.54\, cm$, $1\, mi = 1609\, m$; $1\, N (newton) = kg \cdot m/s^2$, $1\, J (joule) = N \cdot m = kg \cdot m^2/s^2$, $1\, W (watt) = J/s$

Volumes. Cylinder: $\pi R^2 h$, sphere: $\frac{4}{3}\pi R^3$, cone: $\frac{1}{3}\pi R^2 h$

Quadratic equation. $ax^2 + bx + c = 0$, $x = \left(-b \pm \sqrt{b^2-4ac}\right)/(2a)$

Derivatives/integrals. $\frac{d}{dt} t^n = n t^{n-1}$, $\int f^n \, dr = \frac{1}{n+1} t^{n+1}$

Vectors. If $\vec{c} = \vec{a} + \vec{b}$, then $c_x = a_x + b_x$, $c_y = a_y + b_y$, $c_z = a_z + b_z$ and $c = \sqrt{c_x^2 + c_y^2 + c_z^2}$. Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab \cos \alpha$. Cross product: $i \times j = k$, $j \times k = i$, $k \times i = j$.

Kinematics: $v = dx/dt$, $a = dv/dt = d^2x/dt^2$. Constant $a$: $v = v_0 + at$, $x = x_0 + \frac{v_0 + v}{2}t = v_0t + \frac{1}{2}at^2$. Projectile: $v_x = \text{const}$, $x-x_0 = vt$, $y = y_0 - gt$, $y-y_0 = v_0t - \frac{1}{2}gt^2 = (v_{0y}^2 - v_{0y}^2)/(2g)$. Range: $v_0^2/g = \text{sin}(\theta)$

Circular motion with constant speed: $\omega = v/R$, $a_c = v^2/R = \omega^2 R$, towards center.

The three Laws of motion: (1) If $\vec{F}_{net} = 0$ then $\vec{a} = \text{const}$; (2) $\vec{F}_{net} = m\vec{a}$; (3) $\vec{F}_{21} = -\vec{F}_{12}$

Specific forces. Gravity: $mg$ (down). Normal $\vec{N}$ - perpendicular to surface; tension $T$ - constant along the string. Spring force: $F = -kx$ ($k$ is spring constant).

Friction - parallel to surface; kinetic: $f_k = \mu_k N$; static: $f_s \leq \mu_s N$ with $N = mg$ (horizontal plane) or $N = mg \cos \theta$ (inclined plane).

Inclined plane. Components of gravity: $mg \sin \theta$ (parallel to plane, downhill) and $mg \cos \theta$ (perpendicular to plane). Kinetic friction: $\mu_k mg \cos \theta$ (parallel to plane, opposite to direction of motion).

Centripetal motion: $F_{net} = mv^2/R$; direction of $\vec{F}_{net}$ - towards center of revolution.

Work and power. Constant force $W = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1) = F_x \Delta x + F_y \Delta y + F_z \Delta z$ (or, $W = F \Delta r \cos \alpha$); general: $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$. Power: $P = W/\Delta t = \vec{F} \cdot \vec{v}$. Work by specific forces: gravity: $W_g = -mg \Delta y$ (and $\Delta x$ does not matter); normal: $W_N = 0$; kinetic friction: $W_f = -fL$; spring $W_s = \frac{1}{2}k \left( x_1^2 - x_2^2 \right)$.

Kinetic energy and work-energy theorem: $K = \frac{1}{2}mv^2$, $\Delta K = W$ where $W$ is the net work (i.e. work by all forces).

Potential energy. For conservative forces (with path-independent work) introduce $U (\vec{r})$ so that $W_{AB} = U_A - U_B = -\Delta U$. For specific forces: gravity: $U_g = mgh$; spring: $U_s = \frac{1}{2}kx^2$.

If only conservative forces, then energy conservation:

$$K + U = \text{const}$$

If also non-conservative forces (e.g., friction) with work $W_{non-cons}$, then $\Delta (K + U) = W_{non-cons}$

Momentum. $\vec{p} = m\vec{v}$, $\vec{P}_{tot} = \sum m_i \vec{v}_i$. Impulse $\Delta \vec{p} = \vec{P}_{ext} \Delta t$. Conservation: if $\vec{F}_{ext} = 0$, then $\vec{P}_{tot} = \text{const}$ (e.g., in collisions).

Rotation (kinematics): If $N$-number of revolutions, then $\theta = 2\pi N$. Angular velocity $\omega = d\theta/dt$ (in rad/s); ang. acceleration $\alpha = d\omega/dt$ (in rad/s^2). If $\alpha = \text{const}$, then $\omega = \omega_0 t + \frac{1}{2} \alpha t^2$ and $\omega = \omega_0 + at$. Connection with linear: $\omega = v/r$, $\alpha = a/r$

Dynamics: $K = \frac{1}{2}I\omega^2$; I-moment of inertia. For point masses: $I = \sum m_i r_i^2$, for solid bodies $I = \int dV \rho r^2$. Specific I's: rod (about center) $ML^2/12$; rod (about end) $ML^2/3$; hoop $MR^2$; disk $MR^2/2$; solid sphere $\frac{2}{5}MR^2$. Parallel axes theorem: $I = I_{CM} + MD^2$.

Torque $\tau = FR \sin \phi = Fr$. The 2nd Law for rotation: $\tau = I\alpha$.

Rotation+linear: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. If $\omega = v/R$ (e.g., rolling) $K = \frac{1}{2}mv^2(1 + I/(MR^2))$