Formulas - exam 2

Constants and units: 
- \( g = 9.8 \text{ m/s}^2 \)
- \( 1 \text{ mm} = 10^{-3} \text{ m} \)
- \( 1 \text{ cm} = 10^{-2} \text{ m} \)
- \( 1 \text{ km} = 10^3 \text{ m} \)
- \( 1 \text{ in} = 2.54 \text{ cm} \)
- \( 1 \text{ N} = 1 \text{ kg} \cdot \text{ m/s}^2 \)
- \( 1 \text{ J} = 1 \text{ kg} \cdot \text{ m}^2/\text{s}^2 \)
- \( 1 \text{ W} = 1 \text{ Watt} = 1 \text{ J/s} \)

Volumes. 
- Cylinder: \( \pi R^2 h \)
- \( \frac{2}{3} \pi R^3 \)

Quadratic equation. 
\[ ax^2 + bx + c = 0, \quad x = \left( -b \pm \sqrt{b^2 - 4ac} \right) / (2a) \]

Vectors. 
- If \( \vec{a} = a_i \hat{i} + a_j \hat{j} + a_k \hat{k} \), then \( \vec{c} = \vec{a} + \vec{b} \)
- \( c_x = a_x + b_x \), \( c_y = a_y + b_y \), \( c_z = a_z + b_z \)

Kinematics. 
- 1D: \( v = \frac{dx}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \)
- \( \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \)

Circular motion with constant speed: \( \omega = \frac{v}{r}, \quad a_c = \frac{v^2}{r} = \omega^2 r \) towards center.

The three Laws of motion: 
1. If \( F_{\text{net}} = 0 \), then \( \vec{v} = \text{const} \)
2. \( F_{\text{net}} = m \ddot{a} \)
3. \( F_{21} = - F_{12} \)

Specific forces. Gravity: \( mg \) (down). Normal \( N \) - perpendicular to surface; tension \( T \) - constant along the string. Spring force: \( F = -kx \) (k is spring constant).

Friction - parallel to surface; kinetic: \( f_k = \mu_k N \); static: \( f_s \leq \mu_s N \) with \( N = mg \) (horizontal plane) or \( N = mg \cos \theta \) (inclined plane).

Inclined plane. Components of gravity: \( mg \sin \theta \) (parallel to plane, downhill) and \( mg \cos \theta \) (perpendicular to plane).

Centripetal motion: \( F_{\text{net}} = m \omega^2 R \); direction of \( \vec{F}_{\text{net}} \) - towards center of revolution.

Work and power. 
- Constant force \( F = const \) \( W = F \cdot \Delta d \)
- Power: \( P = W / \Delta t = F \cdot \vec{v} \)

Kinetic energy and work-energy theorem: \( K = \frac{1}{2} mv^2 \), \( \Delta K = W \) where \( W \) is the net work (i.e. work by all forces).

Potential energy. 
- For conservative forces (with path-independent work) introduce \( U(r) \) so that \( W_{AB} = U_A - U_B = -\Delta U \).
- For specific forces: gravity: \( U_g = mgh \); spring: \( U_s = \frac{1}{2} k x^2 \).

If only conservative forces, then energy conservation:
\[ K + U = \text{const} \]

If also non-conservative forces (e.g., friction) with work \( W_{\text{non-cons}} \), then \( \Delta (K + U) = W_{\text{non-cons}} \)

Momentum. \( \vec{p} = m \vec{v}, \quad \vec{P}_{\text{tot}} = \sum m_i \vec{v}_i \) . Impulse \( \Delta \vec{p} = \vec{P}_{\text{ext}} \Delta t \). Conservation: if \( \vec{F}_{\text{ext}} = 0 \), then \( \vec{P}_{\text{tot}} = \text{const} \) (e.g., in collisions).

Rotation (kinematics): 
- If \( N \)-number of revolutions, then \( \theta = 2\pi N \). Angular velocity \( \omega = d\theta/dt \) (in rad/s); ang. acceleration \( \alpha = d\omega/dt \) (in rad/s^2).
- If \( \alpha = \text{const} \), then \( \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \) and \( \omega = \omega_0 + \alpha t \). Connection with linear: \( \omega = v/r, \quad \alpha = a/r \)

Dynamics: 
- \( K = \frac{1}{2} I \omega^2 \); \( I \)-moment of inertia.
- For point masses: \( I = \sum m_i r_i^2 \), for solid bodies \( I = \int dV \rho r^2 \), Specific \( I \)'s: rod (about center) \( ML^2/12 \); rod (about end) \( ML^2/3 \); hoop \( MR^2 \); disk \( MR^2/2 \); solid sphere \( \frac{2}{5} MR^2 \). Parallel axes theorem: \( I = I_{CM} + MD^2 \)
- Torque \( \tau = F R \sin \phi = Fd \). The 2nd Law for rotation: \( \tau = I \alpha \).
- Rotation+linear: \( K = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 \). If \( \omega = v/R \) (e.g., rolling) \( K = \frac{1}{2} mv^2(1 + 1/(MR^2)) \)