

Formulas - Final Exam

Constants and units: $g = 9.8 \text{ m/s}^2$, $1 \text{ mm} = 10^{-3} \text{ m}$, $1 \text{ cm} = 10^{-2} \text{ m}$, $1 \text{ km} = 10^3 \text{ m}$, $1 \text{ in} = 2.54 \text{ cm}$, $1 \text{ mi} = 1609 \text{ m}$; $1 \text{ N (newton)} = \text{kg} \cdot \text{m/s}^2$, $1 \text{ J (joule)} = \text{N} \cdot \text{m} = \text{kg} \cdot \text{m}^2/\text{s}^2$, $1 \text{ W (watt)} = \text{J/s}$. density=mass/volume.

Volumes. Cylinder: $\pi R^2 h$, sphere: $\frac{4}{3}\pi R^3$, cone: $\frac{1}{3}\pi R^2 h$

Quadratic equation. $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Derivatives/integrals. $\frac{d}{dt}t^n = nt^{n-1}$, $\int r^n dr = \frac{1}{n+1}r^{n+1}$

Vectors. If $\vec{c} = \vec{a} + \vec{b}$, then $c_x = a_x + b_x$, $c_y = a_y + b_y$, $c_z = a_z + b_z$ and $c = \sqrt{c_x^2 + c_y^2 + c_z^2}$.

Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab \cos \alpha$. Cross product: $|\vec{A} \times \vec{B}| = A * B * \sin \alpha$, $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.

Kinematics: $v = dx/dt$, $a = dv/dt = d^2x/dt^2$. Constant a : $v - v_0 = at$, $x - x_0 = \frac{v_0 + v}{2}t = v_0 t + \frac{1}{2}at^2 = \frac{v^2 - v_0^2}{2a}$. Projectile: $v_x = \text{const}$, $x - x_0 = v_x t$, $v_y = v_{0y} - gt$, $y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = (v_{0y}^2 - v_y^2)/(2g)$. Range: $(v_0^2/g) * \sin(2\theta)$

Circular motion with constant speed: $\omega = v/R$, $a_c = v^2/R = \omega^2 R$, towards center.

The three Laws of motion: (1) If $\vec{F}_{net} = 0$ then $\vec{v} = \text{const}$; (2) $\vec{F}_{net} = m\vec{a}$; (3) $\vec{F}_{21} = -\vec{F}_{12}$
Specific forces. Gravity: $m\vec{g}$ (down). Normal \vec{N} - perpendicular to surface; tension T - constant along the string. Spring force: $F = -kx$ (k is spring constant).

Friction - parallel to surface; kinetic: $f_k = \mu_k N$; static: $f_s \leq \mu_s N$ with $N = mg$ (horizontal plane) or $N = mg \cos \theta$ (inclined plane).

Inclined plane. Components of gravity: $mg \sin \theta$ (parallel to plane, downhill) and $mg \cos \theta$ (perpendicular to plane). Kinetic friction: $\mu_k mg \cos \theta$ (parallel to plane, opposite to direction of motion).

Centripetal motion: $F_{net} = mv^2/R$; direction of \vec{F}_{net} - towards center of revolution.

Work and power. Constant force $W = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1) = F_x \Delta x + F_y \Delta y + F_z \Delta z$ (or, $W = F \Delta r \cos \alpha$); general: $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$. Power: $P = W/\Delta t = \vec{F} \cdot \vec{v}$. Work by specific forces: gravity: $W_g = -mg \Delta y$ (and Δx does not matter); normal: $W_N = 0$;

kinetic friction: $W_f = -fL$; spring $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$

Kinetic energy and work-energy theorem: $K = \frac{1}{2}mv^2$, $\Delta K = W$ where W is the net work (i.e. work by all forces).

Potential energy. For conservative forces (with path-independent work) introduce $U(\vec{r})$ so that $W_{AB} = U_A - U_B = -\Delta U$. For specific forces: gravity: $U_g = mgh$; spring: $U_s = \frac{1}{2}kx^2$. If only conservative forces, then energy conservation: $\Delta K + \Delta U = 0$ or $K + U = \text{const}$. If also non-conservative forces (e.g., friction) with work $W_{non-cons}$, then $\Delta(K + U) = W_{non-cons}$

Momentum. $\vec{p} = m\vec{v}$, $\vec{P}_{tot} = \sum m_i \vec{v}_i$. Impulse $\Delta \vec{p} = \vec{F}_{av} \Delta t$. Conservation: if $\vec{F}_{ext} = 0$, then $\vec{P}_{tot} = \text{const}$ (e.g., in collisions). **Center-of-mass:** $\vec{R}_{cm} = \frac{1}{M} \sum m_i \vec{r}_i$, $M = \sum M_i$; $\vec{P} = M\vec{V}_{cm}$

Rotation (kinematics): If N -number of revolutions, then $\theta = 2\pi N$. Angular velocity $\omega = d\theta/dt$ (in rad/s); ang. acceleration $\alpha = d\omega/dt$ (in rad/s²). If $\alpha = \text{const}$, then $\theta = \frac{\omega_0 + \omega}{2}t = \omega_0 t + \frac{1}{2}\alpha t^2$, or $\theta = \frac{\omega^2 - \omega_0^2}{2\alpha}$, and $\omega = \omega_0 + \alpha t$. Connection with linear: $\omega = v/r$, $\alpha = a/r$

Dynamics: $K = \frac{1}{2}I\omega^2$; I -moment of inertia. For point masses: $I = \sum m_i r_i^2$, for solid bodies $I = \int dV \rho r^2$. Specific I 's: rod (about center) $ML^2/12$; rod (about end) $ML^2/3$; hoop MR^2 ; disk $MR^2/2$; solid sphere $\frac{2}{5}MR^2$; spherical shell $\frac{2}{3}MR^2$. Parallel axes theorem: $I = I_{CM} + MD^2$

Rotation+linear: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. If $\omega = v/R$ (e.g., rolling) $K = \frac{1}{2}mv^2(1 + I/(mR^2))$

Torque $\vec{\tau} = \vec{r} \times \vec{F}$, $\tau = FR \sin \phi = Fd$. The 2nd Law for rotation: $\tau = I\alpha$.

Angular momentum. $\vec{L} = \sum \vec{r}_i \times \vec{p}_i$ (point masses); $L = I\omega$ (symmetric solid). Conservation: if $\vec{\tau} = 0$ then $\vec{L} = \text{const}$.

Static equilibrium. $\sum \vec{F}_i = 0$, $\sum \vec{\tau}_i = 0$.

Fluids. Statics. $P = F_n/A$, $P = \rho gh$ (barostatic equation), $F_B = \rho_{\text{fluid}} g V_{\text{submerged}}$ (Archimedes Law).

Fluids. Dynamics. Continuity equation: $\rho v A = \text{const}$.

Bernoulli equation: $P + \rho gh + \frac{1}{2}\rho v^2 = \text{const}$.

Gravity. $F = -GMm/r^2$, $g(r) = -GM/r^2$. $U(r) = -GMm/r$. 3rd Law of Kepler: $T^2 \propto r^3$. $T = (2\pi/\sqrt{GM})r^{3/2}$. Satellite: $v_{orb} = \sqrt{g(r) * r}$, r -distance from center of planet (for low orbits $r \simeq R_p$); $g(r) = g_s * (R_p/r)^2$. Escape: $v_{esc} = \sqrt{2}v_{orb} = \sqrt{2GM_p/R_p}$ (from surface of planet). $G = 6.67 * 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$. $M_{\text{sun}} = 1.99 * 10^{30} \text{ kg}$; $M_{\text{earth}} = 5.97 * 10^{24} \text{ kg}$; distance from Sun to Earth $150 * 10^6 \text{ km}$.