

Formulas PHYS 121- exam2

Units: SI system: kg (kilogram), m (meter), s (second), C (coulomb); \mathcal{A} (ampere)=C/s, V (volt)=J/C, F (farad)=C/V, Ω (ohm)=V/ \mathcal{A} . 1 mm = 10^{-3} m, 1 cm = 10^{-2} m, 1 km = 10^3 m; 1 N (newton)=kg·m/s², 1 J (joule)=N·m = kg·m²/s², 1 W (watt)=J/s; prefixes: m (milli) 10^{-3} , μ (micro) 10^{-6} , n (nano) 10^{-9} , p(pico) 10^{-12} , k (kilo) 10^3 , M (mega) 10^6 .

Constants: $g = 9.8 \text{ m/s}^2$, $k_e = 9 * 10^9 \text{ N m}^2/\text{C}^2 = 1/(4\pi\epsilon_0)$, $\epsilon_0 = 8.85 * 10^{-12} \text{ C}^2/(\text{N m}^2)$. $e = -1.6 * 10^{-19} \text{ C}$, $m_e = 9.11 * 10^{-31} \text{ kg}$, $m_p \simeq m_n = 1.67 * 10^{-27} \text{ kg}$

Volumes. Cylinder: $\pi R^2 h$, sphere: $\frac{4}{3}\pi R^3$, cone: $\frac{1}{3}\pi R^2 h$. **Areas:** Sphere: $4\pi R^2$

Quadratic equation. $ax^2 + bx + c = 0$, $x = \left(-b \pm \sqrt{b^2 - 4ac} \right) / (2a)$

Derivatives/integrals. $\frac{d}{dt}t^n = nt^{n-1}$, $\int r^n dr = \frac{1}{n+1}r^{n+1}$

Vectors. If $\vec{c} = \vec{a} + \vec{b}$, then $c_x = a_x + b_x$, $c_y = a_y + b_y$, $c_z = a_z + b_z$ and $c = \sqrt{c_x^2 + c_y^2 + c_z^2}$. Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab \cos \alpha$. Cross product: $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$; $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$.

Coulombs Law: $F = k_e \frac{q_1 q_2}{r^2}$, r -distance between charges; in vector form $\vec{F} = k_e \frac{q_1 q_2}{r^2} \hat{r}$, $\hat{r} = \vec{r}/r$ - unit vector from charge q_1 to q_2 , $k_e = 9 * 10^9 \dots$ Superposition: if charge q_1 acts on q_0 with \vec{F}_{01} , charge q_2 acts on q_0 with \vec{F}_{02} , etc., then $\vec{F}_{\text{net on } q_0} = \vec{F}_{01} + \vec{F}_{02} + \dots$

Electric field. Definition: $\vec{E} = \vec{F}_0/q_0$ (charge q_0 is a "probe"). Field from a charge q : $E = k_e \frac{q}{r^2}$, r -distance between charge and observation point; in vector form $\vec{E} = k_e \frac{q}{r^2} \hat{r}$, $\hat{r} = \vec{r}/r$ - unit vector from charge q to the observation point, $k_e = 9 * 10^9 \dots$ Superposition: consider charges q_1, q_2 , etc. and the observation point. If q_1 creates field \vec{E}_1 at the observation point, q_2 creates field \vec{E}_2 , etc., then $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$
Force on a charge placed in external field \vec{E} : $\vec{F} = q\vec{E}$.

Gauss Law. Flux through a small area A with \vec{A} along the normal to the surface: $\Phi = \vec{E} \cdot \vec{A}$. Flux through a closed surface: $\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\epsilon_0$. Field $E(r)$ from a uniformly charged spherical shell with radius R and charge Q : $E(r < R) = 0$, $E(r > R) = k_e Q/r^2$. Field $E(r)$ from a uniformly charged infinite line with linear charge density λ : $E(r) = \lambda/(2\pi\epsilon_0 r)$. Field E from a uniformly charged infinite non-conducting plane with surface charge density σ : $E = \sigma/(2\epsilon_0)$.

Potential. Definition: $V = U/q_0$. Work: $W_{AB} = U_A - U_B = -q\Delta V$. Two charges: $U = k_e q_1 q_2 / r$. Point charge: $V(r) = k_e q / r$. Superposition: $V(\vec{r}) = k_e (q_1/r_1 + q_2/r_2 + \dots)$ (with r_1 - distance from q_1 to observation point, etc.). Potential from field: $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$; for uniform field: $\Delta V = -Ed$. Field from potential: $E_x = -dV/dx, \dots$ Conducting sphere with charge Q and radius R : $V(r) = k_e Q/R$, $r \geq R$ and $V(r) = kQ/R$, $r \leq R$.

Conductors. Inside the body of a conductor: $\vec{E} = 0$, $V = \text{const}$, no charge. Extra charge - goes to outer surface. Field near the surface of a conductor, *outside*, is $E = \sigma/\epsilon_0$; potential - continuous. Inner surfaces (surfaces of cavities) - uncharged, unless an extra charge is placed inside a cavity.

Capacitors. Definition: $C = Q/V$. Single sphere: $C = 4\pi\epsilon_0 R$. Spherical: $C = 4\pi\epsilon_0 R_1 R_2 / (R_2 - R_1)$. Parallel plate capacitor: $C = A\epsilon_0 \kappa / d$. Energy: $U_C = Q^2 / (2C) = V^2 C / 2$. Connections: parallel (same voltage) $C_{eq} = C_1 + C_2 + \dots$; series (same charge) $1/C_{eq} = 1/C_1 + 1/C_2 + \dots$ or $C_{eq} = C_1 C_2 / (C_1 + C_2)$ (for two only). $Q_{\text{tot}} = V C_{eq}$.

Current. Definitions: current $I = \Delta q / \Delta t \simeq dq / dt$, density of current $J = I / A$, with A - cross-sectional area. Ohm's law: $I = V / R$ with R -resistance. For wire of length L : $R = \rho L / A$, with ρ - resistivity of material. $J = E / \rho$. Power: $P = IV = I^2 R = V^2 / R$. Simple connections: series (same current) $R_{eq} = R_1 + R_2 + \dots$; parallel (same voltage) $1 / R_{eq} = 1 / R_1 + 1 / R_2 + \dots$ or $R_{eq} = R_1 R_2 / (R_1 + R_2)$ (for two only). Microscopic picture: $J = env_d$.

From Phys 111:

Kinematics: $v = dx / dt$, $a = dv / dt = d^2 x / dt^2$. Constant a : $v - v_0 = at$, $x - x_0 = \frac{v_0 + v}{2} t = v_0 t + \frac{1}{2} at^2 = \frac{v^2 - v_0^2}{2a}$.

Circular motion with constant speed: $\omega = v / R$, $a_c = v^2 / R = \omega^2 R$, towards center.

The three Laws of motion: (1) If $\vec{F}_{net} = 0$ then $\vec{v} = const$; (2) $\vec{F}_{net} = m\vec{a}$; (3) $\vec{F}_{21} = -\vec{F}_{12}$
 Specific forces. Gravity: $m\vec{g}$ (down). Normal \vec{N} - perpendicular to surface; tension T - constant along the string.

Centripetal motion: $F_{net} = mv^2 / R$; direction of \vec{F}_{net} - towards center of revolution.

Work and power. Constant force $W = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1) = F_x \Delta x + F_y \Delta y + F_z \Delta z$ (or, $W = F \Delta r \cos \alpha$); general: $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$. Power: $P = W / \Delta t = \vec{F} \cdot \vec{v}$. Work by specific forces: gravity: $W_g = -mg \Delta y$ (and Δx does not matter); normal: $W_N = 0$.

Kinetic energy and work-energy theorem: $K = \frac{1}{2} mv^2$, $\Delta K = W$ where W is the *net* work (i.e. work by all forces).

Potential energy. For conservative forces (with path-independent work) introduce $U(\vec{r})$ so that $W_{AB} = U_A - U_B = -\Delta U$. For specific forces: gravity: $U_g = mgh$. If *only* conservative forces, then energy conservation: $K + U = const$.