Units: SI system: kg (kilogram), m (meter), s (second), C (coulomb); A (ampere)=C/s, V (volt)=J/C, F (farad)=C/V, Ω (ohm)=V/A. 1 mm = 10^{-3} m, 1 cm = 10^{-2} m, 1 km = 10^3 m; 1 N (newton)=kg·m/s^2, 1 J (joule)=N·m = kg·m^2/s^2, 1 W (watt)=J/s; prefixes: m (milli) 10^{-3}, µ (micro) 10^{-6}, n (nano) 10^{-9}, p(pico) 10^{-12}, k (kilo) 10^3, M (mega) 10^6.

Constants: g = 9.8 m/s^2, k_e = 9 * 10^9 N m^2/C^2 = 1/(4πε_0), ε_0 = 8.85 * 10^{-12} C^2/(N m^2), µ_0 = 4π*10^{-7} T⋅m/A. e = -1.6 * 10^{-19} C, m_e = 9.11 * 10^{-31} kg, m_p = m_e = 1.67 * 10^{-27} kg


Quadratic equation. ax^2 + bx + c = 0, x = (-b ± √b^2 - 4ac) / (2a)

Derivatives/integrals. d/dx x^n = nx^n−1, d/dx sin x = cos x, d/dx cos x = −sin x, d/dx e^{ax} = ae^{ax}; ∫x^n dx = 1/n+1 x^{n+1}, ∫dx/x = ln x; ∫dx/√a^2 + x^2 = ln(x + √a^2 + x^2);

∫dx (a^2 + x^2)^−3/2 = x/(a^2√a^2 + x^2); ∫dx (a^2 + x^2)^−2 = −1/(√a^2 + x^2); ∫sin x dx = −cos x; ∫cos x dx = sin x.

Vectors. If c = a + b, then c_x = a_x + b_x, c_y = a_y + b_y, c_z = a_z + b_z and c = √c_x^2 + c_y^2 + c_z^2. Dot product: a ⋅ b = a_xb_x + a_yb_y + a_zb_z = a*b*cos θ. Cross product: i ⨯ j = k, j ⨯ i = k; A ⨯ B = (AyBz - AzBy)i + (AzBx - AxBy)j + (AxBy - AyBz)k; A ⨯ B = A*B*sin α.

Coulomb’s Law: F = k_e|q_1q_2|r, r-distance between charges; in vector form \( \vec{F}_1 = k_e \frac{q_1q_2}{r^2} \hat{r} \), \( \hat{r} = \hat{r}/r \) - unit vector from charge q_1 to q_2, \( k_e = 9 * 10^9 \ldots \) Superposition: if charge q_1 acts on q_0 with \( \vec{F}_{q_1} \), charge q_2 acts on q_0 with \( \vec{F}_{q_2} \), etc., then \( \vec{F}_{\text{net}} = \vec{F}_{q_1} + \vec{F}_{q_2} + \ldots \)

Electric field. Definition: \( \vec{E} = \vec{F}_0 / q_0 \) (charge q_0 is a ”probe”). Field from a charge q: \( \vec{E} = k_e \frac{q}{|q|^2} \hat{r} \), r-distance between charge and observation point; in vector form \( \vec{E} = k_e \frac{q}{|q|^2} \hat{r} \), \( \hat{r} = \hat{r}/r \) - unit vector from charge q to the observation point, \( k_e = 9 * 10^9 \ldots \) Superposition: consider charges q_1, q_2, etc. and the observation point. If q_1 creates field \( \vec{E}_1 \) at the observation point, q_2 creates field \( \vec{E}_2 \), etc., then \( \vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots \)

Force on a charge placed in external field \( \vec{E} \): \( \vec{F} = q\vec{E} \).

Gauss Law. Flux through a small area \( A \) with \( \vec{A} \) along the normal to the surface: \( \Phi = \vec{E} \cdot \vec{A} \). Flux through a closed surface: \( \oint \vec{E} \cdot d\vec{S} = q_{\text{enc}}/\epsilon_0 \). Field \( E(r) \) from a uniformly charged spherical shell with radius R and charge Q: \( E(r < R) = 0, E(r > R) = k_eQ/r^2 \). Field \( E(r) \) from a uniformly charged infinite line with linear charge density \( \lambda \): \( E(r) = \lambda/(2\pi\epsilon_0r) = 2k_e\lambda/r \). Field \( E \) from a uniformly charged infinite non-conducting plane with surface charge density \( \sigma \): \( E = \sigma/(2\epsilon_0) = 2\pi k_e \sigma \). Field near a conductor: \( E = \sigma/\epsilon_0 \).

Potential. Definition: \( V = U/q_0 \). Work: \( W_{AB} = U_A - U_B = -q\Delta V \). Two charges: \( U = k_e|q_1q_2|/r \). Point charge: \( V(r) = k_eq/r \). Superposition: \( V(r) = k_e(q_1/r_1 + q_2/r_2 + \ldots) \) (with \( r_1 \) - distance from q_1 to observation point, etc.). Potential from field: \( V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{S} \) for uniform field: \( \Delta V = -Ed \). Field from potential: \( E_x = -dV/dx, \ldots \) Conducting sphere with charge Q and radius R: \( V(r) = k_eQ/r, r \geq R \) and \( V(r) = k_eQ/R, r \leq R \).

Conductors. Inside the body of a conductor: \( \vec{E} = 0 \), \( V = \text{const}, \) no charge. Extra charge - goes to outer surface. Field near the surface of a conductor, outside, is \( E = \sigma/\epsilon_0 \); potential - continuous. Inner surfaces (surfaces of cavities) - uncharged, unless an extra charge is placed inside a cavity.
**Capacitors.** Definition: \( C = Q/V \). Single sphere: \( C = 4\pi\varepsilon_0 R \). Spherical: \( C = \varepsilon_0\kappa/R \). Parallel plate capacitor: \( C = \varepsilon_0\kappa/d \). Energy: \( U_C = \frac{Q^2}{2C} = \frac{V^2}{2} \). Connections: parallel (same voltage) \( C_{eq} = C_1 + C_2 + \ldots \); series (same charge) \( 1/C_{eq} = 1/C_1 + 1/C_2 + \ldots \) or \( C_{eq} = C_1C_2/(C_1 + C_2) \) (for two only). \( Q_{tot} = VC_{eq} \).

**Current.** Definitions: current \( I = \Delta q/\Delta t \simeq dq/dt \), density of current \( J = I/A \), with \( A \) - cross-sectional area. Ohm’s law: \( I = VR \) with \( R \)-resistance. For wire of length \( L \): \( R = \rho L/A \), with \( \rho \) - resistivity of material. \( J = E/\rho \). Power: \( P = IV = I^2R = V^2/R \).

Simple connections: series (same current) \( R_{eq} = R_1 + R_2 + \ldots \); parallel (same voltage) \( \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \) or \( R_{eq} = R_1R_2/(R_1 + R_2) \) (for two only). Microscopic picture: \( J = \text{env} \).

**Multiloop circuits and Kirchoff’s equations.** Potential changes: \( +E \) when crossing battery from negative to positive terminal; \( -IR \) when traveling through a resistor along the current. Loop equation: potential drop around a close loop is zero. Junction equation: sum of incoming currents equals sum of outgoing currents.

Charging a capacitor: \( q(t) = EC(1 - e^{-t/\tau}) \) with \( \tau = RC \); \( V_C(t) = \frac{q}{C} = E(1 - e^{-t/\tau}) \), \( i(t) = i_{max}e^{-t/\tau} \) with \( i_{max} = \frac{Q_0}{RC} \). Discharging a capacitor: \( \tau = RC \); \( q(t) = Q_0e^{-t/\tau} \), \( V(t) = V_0e^{-t/\tau} \) with \( V_0 = \frac{Q_0}{C} \); \( i(t) = i_0e^{-t/\tau} \) with \( i_0 = \frac{Q_0}{RC} \).

From Phys 111:

**Kinematics:** \( v = dx/dt \), \( a = dv/dt = d^2x/dt^2 \). Circular motion with constant speed: \( \omega = v/R \), \( a_c = v^2/R = \omega^2R \), towards center.

**The three Laws of motion:** (1) If \( \vec{F}_{net} = 0 \) then \( \vec{v} = \text{const} \); (2) \( \vec{F}_{net} = m\vec{a} \); (3) \( \vec{F}_{21} = -\vec{F}_{12} \)

**Work and power.** \( W_{AB} = \int_A^B \vec{F} \cdot d\vec{r} \). Power: \( P = W/\Delta t = \vec{F} \cdot \vec{v} \).

**Kinetic energy and work-energy theorem:** \( K = \frac{1}{2}mv^2 \), \( \Delta K = W \) where \( W \) is the net work (i.e. work by all forces).

**Potential energy.** For conservative forces (with path-independent work) introduce \( U(\vec{r}) \) so that \( W_{AB} = U_A - U_B = -\Delta U \). If only conservative forces, then energy conservation: \( K + U = \text{const} \).