Formulas PHYS 121- exam3

Units: SI system: kg (kilogram), m (meter), s (second), C (coulomb); $\mathcal{A}$ (ampere)=C/s, V (volt)=J/C, F (farad)=C/V, Ω (ohm)=V/$\mathcal{A}$. $1\,\text{mm} = 10^{-3}\,\text{m}$, $1\,\text{cm} = 10^{-2}\,\text{m}$, $1\,\text{km} = 10^{3}\,\text{m}$; 1 N (newton)=kg·m/s^2, 1 J (joule)=N $\cdot$ m = kg $\cdot$ m^2/s^2, 1 W (watt)=J/s; prefixes: m (milli) 10^{-3}, μ (micro) 10^{-6}, n (nano) 10^{-9}, p(pico) 10^{-12}, k (kilo) 10^3, M (mega) 10^6.
Constants: $g = 9.8\,\text{m/s}^2$, $k_e = 9 * 10^9\,\text{N m}^2/\text{C}^2 = 1/(4\pi\epsilon_0)$, $\epsilon_0 = 8.85 * 10^{-12}\,\text{C}^2/(\text{N m}^2)$, $\mu_0 = 4\pi * 10^{-7}\, T{-}m/A$. $e = -1.6 * 10^{-19}\,\text{C}$, $m_e = 9.11 * 10^{-31}\,\text{kg}$, $m_p \simeq m_n = 1.67 * 10^{-27}\, kg$
Volumes. Cylinder: $\pi R^2 h$, sphere: $\frac{4}{3}\pi R^3$, cone: $\frac{1}{3}\pi R^2 h$. **Areas:** Sphere: $4\pi R^2$
Quadratic equation. $ax^2 + bx + c = 0\,,\ x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/(2a)$
Derivatives/integrals. $\frac{d}{dt}t^n = nt^{n-1}\,,\ \int r^n\,dr = \frac{1}{n+1}r^{n+1}$
Vectors. If $\vec{c} = \vec{a} + \vec{b}$, then $c_x = a_x + b_x\,,\ c_y = a_y + b_y\,,\ c_z = a_z + b_z$ and $c = \sqrt{c_x^2 + c_y^2 + c_z^2}$. Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\alpha$. Cross product: $\hat{\imath} \times \hat{\jmath} = \hat{k}\,,\ \hat{\jmath} \times \hat{k} = \hat{\imath}\,,\ \hat{k} \times \hat{\imath} = \hat{\jmath}$; $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$.

Coulombs Law: $F = k_e \frac{q_1 q_2}{r^2}$, r-distance between charges; in vector form $\vec{F} = k_e \frac{q_1 q_2}{r^2}\hat{r}$, $\hat{r} = \vec{r}/r$ - unit vector from charge q_1 to q_2, $k_e = 9 * 10^9$... Superposition: if charge q_1 acts on q_0 with $\vec{F}_{01}$, charge q_2 acts on q_0 with $\vec{F}_{02}$, etc., then $\vec{F}_{\text{net on } q_0} = \vec{F}_{01} + \vec{F}_{02} + \ldots$

Electric field. Definition: $\vec{E} = \vec{F}_0/q_0$ (charge q_0 is a "probe"). Field from a charge q: $E = k_e \frac{q}{r^2}$, r-distance between charge and observation point; in vector form $\vec{E} = k_e \frac{q}{r^2}\hat{r}$, $\hat{r} = \vec{r}/r$ - unit vector from charge q to the observation point, $k_e = 9 * 10^9$... Superposition: consider charges q_1, q_2, etc. and the observation point. If q_1 creates field $\vec{E}_1$ at the observation point, q_2 creates field $\vec{E}_2$, etc., then $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$.
Force on a charge placed in external field $\vec{E}$: $\vec{F} = q\vec{E}$.

Gauss Law. Flux through a small area A with $\vec{A}$ along the normal to the surface: $\Phi = \vec{E} \cdot \vec{A}$. Flux through a closed surface: $\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\epsilon_0$. Field $E(r)$ from a uniformly charged spherical shell with radius R and charge Q: $E(r < R) = 0$, $E(r > R) = k_e Q/r^2$. Field $E(r)$ from a uniformly charged infinite line with linear charge density λ: $E(r) = \lambda/(2\pi\epsilon_0 r)$. Field E from a uniformly charged infinite non-conducting plane with surface charge density σ: $E = \sigma/(2\epsilon_0)$.

Potential. Definition: $V = U/q_0$. Work: $W_{AB} = U_A - U_B = -q\Delta V$. Two charges: $U = k_e q_1 q_2/r$. Point charge: $V(r) = k_e q/r$. Superposition: $V(\vec{r}) = k_e\left(q_1/r_1 + q_2/r_2 + \ldots\right)$ (with r_1 - distance from q_1 to observation point, etc.). Potential from field: $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$; for uniform field: $\Delta V = -Ed$. Field from potential: $E_x = -dV/dx, \ldots$. Conducting sphere with charge Q and radius R: $V(r) = k_e Q/r$, $r \geq R$ and $V(r) = kQ/R$, $r \leq R$.

Conductors. Inside the body of a conductor: $\vec{E} = 0$, V =const, no charge. Extra charge - goes to outer surface. Field near the surface of a conductor, *outside*, is $E = \sigma/\epsilon_0$; potential - continuous. Inner surfaces (surfaces of cavities) - uncharged, unless an extra charge is placed inside a cavity.

Capacitors. Definition: $C = Q/V$. Single sphere: $C = 4\pi\epsilon_0 R$. Spherical: $C = 4\pi\epsilon_0 R_1 R_2/(R_2 - R_1)$. Parallel plate capacitor: $C = A\epsilon_0\kappa/d$. Energy: $U_C = Q^2/(2C) = V^2 C/2$. Connections: parallel (same voltage) $C_{eq} = C_1 + C_2 + \ldots$; series (same charge) $1/C_{eq} = 1/C_1 + 1/C_2 + \ldots$ or $C_{eq} = C_1 C_2/(C_1 + C_2)$ (for two only). $Q_{\text{tot}} = VC_{eq}$.

Current. Definitions: current $I = \Delta q/\Delta t \simeq dq/dt$, density of current $J = I/A$, with A - cross-sectional area. Ohm's law: $I = V/R$ with R-resistance. For wire of length L: $R = \rho L/A$, with ρ - resistivity of material. $J = E/\rho$. Power: $P = IV = I^2R = V^2/R$. Simple connections: series (same current) $R_{eq} = R_1 + R_2 + \ldots$; parallel (same voltage) $1/R_{eq} = 1/R_1 + 1/R_2 + \ldots$ or $R_{eq} = R_1R_2/(R_1 + R_2)$ (for two only). Microscopic picture: $J = env_d$.

Multiloop circuits and Kirchoff's equations. Potential changes: $+\mathcal{E}$ when crossing battery from negative to positive terminal; $-IR$ when traveling through a resistor along the current. Loop equation: potential drop around a close loop is zero. Junction equation: sum of incoming currents equals sum of outgoing currents.
Charging a capacitor: $q(t) = \mathcal{E}C\left(1 - e^{-t/\tau}\right)$ with $\tau = RC$; $V_C(t) = \frac{q}{C} = \mathcal{E}\left(1 - e^{-t/\tau}\right)$, $i(t) = i_{\max}e^{-t/\tau}$ with $i_{\max} = \frac{\mathcal{E}}{R}$. Discharging a capacitor: $\tau = RC$, $q(t) = Q_0e^{-t/\tau}$, $V(t) = V_0e^{-t/\tau}$ with $V_0 = \frac{Q_0}{C}$; $i(t) = i_0e^{-t/\tau}$ with $i_0 = \frac{Q_0}{RC}$.

Magnetic force. Force on a particle: $\vec{F}_m = q\vec{v} \times \vec{B}$. Revolution in magnetic field: radius $r = mv/(qB)$, period $T = 2\pi m/(qB)$. Force on a wire: $\vec{F}_w = I\vec{L} \times \vec{B}$. Interaction between two parallel wires: $F = \mu_0 I_1 I_2 L/(2\pi d)$ with $\mu_0 = 4\pi * 10^{-7}$.

Fields from currents. Straight wire: $B = \mu_0 I/(2\pi d)$. Bio-Savarat: $d\vec{B} = (\mu_0/4\pi)Id\vec{s} \times \vec{r}/r^3$. At the center of ring current of radius R: $B_{\text{ring}} = \mu_0 I/(2R)$; for a circular arc with angle θ (in radians): $B = B_{\text{ring}} \times \theta/(2\pi)$. Amper's Law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enc}}$. Field in a solenoid: $B = \mu_0 In$, with n - density of turns.

From Phys 111:
Kinematics: $v = dx/dt$, $a = dv/dt = d^2x/dt^2$. Constant a: $v - v_0 = at$, $x - x_0 = \frac{v_0+v}{2}t = v_0t + \frac{1}{2}at^2 = \frac{v^2-v_0^2}{2a}$.
Circular motion with constant speed: $\omega = v/R$, $a_c = v^2/R = \omega^2 R$, towards center.
The three Laws of motion: (1) If $\vec{F}_{net} = 0$ then $\vec{v} = const$; (2) $\vec{F}_{net} = m\vec{a}$; (3) $\vec{F}_{21} = -\vec{F}_{12}$
Specific forces. Gravity: $m\vec{g}$ (down). Normal $\vec{N}$ - perpendicular to surface; tension T - constant along the string.
Centripetal motion: $F_{net} = mv^2/R$; direction of $\vec{F}_{net}$ - towards center of revolution.
Work and power. Constant force $W = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1) = F_x\Delta x + F_y\Delta y + F_z\Delta z$ (or, $W = F\Delta r\cos\alpha$); general: $W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$. Power: $P = W/\Delta t = \vec{F} \cdot \vec{v}$. Work by specific forces: gravity: $W_g = -mg\Delta y$ (and Δx does not matter); normal: $W_N = 0$.
Kinetic energy and work-energy theorem: $K = \frac{1}{2}mv^2$, $\Delta K = W$ where W is the *net* work (i.e. work by all forces).
Potential energy. For conservative forces (with path-independent work) introduce $U(\vec{r})$ so that $W_{AB} = U_A - U_B = -\Delta U$. For specific forces: gravity: $U_g = mgh$. If *only* conservative forces, then energy conservation: $K + U = const$.